Comparative Valuation Dynamics in Models with Financing Frictions

III. Model Comparisons

Today's Lecture:
Fabrice Tourre (Copenhagen Business School)

Based on joint work with:
Lars Peter Hansen (University of Chicago)
Paymon Khorrami (University of Chicago)

Numerical implementation:
Joseph Huang (University of Chicago)

March 21, 2019

Recap of Previous Lectures

1. Continuous-time recursive utility
2. Complete markets production model with long run risk
3. “Shock elasticities” as model diagnostics
4. Heterogeneous agents, financial frictions, and long run risk
5. Numerical methods

Today's plan: model comparisons

1. Nesting Model – Refresher
2. Binding Constraints and Risk Aversion Heterogeneity
3. Impact of Frictions on Equilibrium Outcomes
4. Long Run Risk and Financial Frictions
5. Long Run Risk and Capital Misallocation

Part I

Nesting Model – Quick Reminder

Nesting Model

Agent Types: “Households” and “Experts”

Technology

- A-K production function with $a_e \geq a_h$
- agg. and idio. TFP shocks (also called “capital quality shocks”)
- agg. growth rate and agg. stochastic vol shocks (long-run risk)

Markets

- Capital traded (with shorting constraint)
- Complete financial markets for households
- Experts facing minimum risk-retention constraint

Preferences

- Recursive utility
- Households and experts potentially different
- OLG (for stationary equilibrium)

Technology

$$\frac{dK_t}{K_t} = \Big[\underbrace{\Phi(I_t/K_t)}_{\text{endogenous growth}} + \underbrace{Z_t - \alpha_k}_{\text{exogenous growth}} \Big] dt + \underbrace{\sqrt{V_t}\sigma_k \cdot dB_t}_{\text{aggregate shocks}} + \underbrace{\sqrt{\tilde{V}_t}\tilde{\sigma}_k d\tilde{B}_t}_{\text{idiosyncratic shocks}}$$

$$\begin{aligned}
\text{(exogenous growth)} \quad & dZ_t = -\lambda_z Z_t dt + \sqrt{V_t}\sigma_z \cdot dB_t \\
\text{(aggregate variance)} \quad & dV_t = -\lambda_v (V_t - 1) dt + \sqrt{V_t}\sigma_v \cdot dB_t \\
\text{(idiosyncratic variance)} \quad & d\tilde{V}_t = -\lambda_{\tilde{v}} (\tilde{V}_t - 1) dt + \sqrt{\tilde{V}_t}\sigma_{\tilde{v}} \cdot dB_t
\end{aligned}$$

$I_t dt$ invested leads to $\Phi\left(I_t/K_t\right) K_t dt$ increase in the capital stock

Markets

Capital is freely traded, at price Q_t

$$dQ_t = Q_t \left[\mu_{q,t} dt + \sigma_{q,t} \cdot dB_t\right]$$

Households facing dynamically complete markets, leading to SDF $S_{h,t}$

$$dS_{h,t} = -S_{h,t} \left[r_t dt + \pi_{h,t} \cdot dB_t\right]$$

Experts face skin-in-the-game constraint via minimum risk retention:

$$\chi_t \geq \underline{\chi}$$

χ_t is fraction of equity retained by experts

Experts SDF $S_{e,t}$

$$dS_{e,t} = -S_{e,t} \left[r_t dt + \pi_{e,t} \cdot dB_t\right]$$

Stochastic Control Problem

Agent i will solve the following problem:

$$U_{i,t} = \max_{\{K_i \geq 0, C_i, \theta_i, i_i\}} \mathbb{E}\left[\int_t^{+\infty} \varphi\left(C_{i,s}, U_{i,s}\right) ds\right]$$

$$\text{s.t. } \frac{dN_{i,t}}{N_{i,t}} = \left[\mu_{n,i,t} - \frac{C_{i,t}}{N_{i,t}}\right] dt + \sigma_{n,i,t} \cdot dB_t + \tilde{\sigma}_{n,i,t} \cdot d\tilde{B}_t$$

$$\mu_{n,i,t} = r_t + \frac{Q_t K_{i,t}}{N_{i,t}} \left(\mu_{R,i,t} - r_t\right) + \theta_{i,t} \cdot \pi_t$$

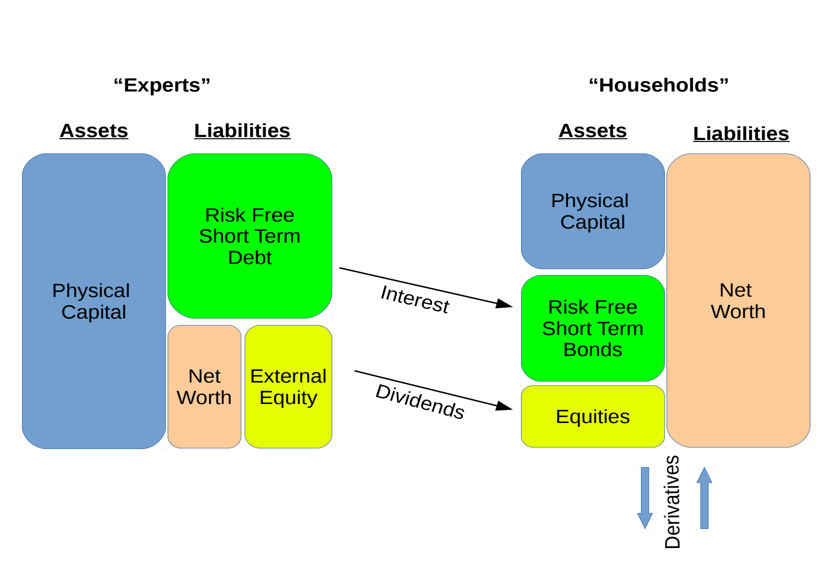
$$\sigma_{n,i,t} = \frac{Q_t K_{i,t}}{N_{i,t}} \sigma_{R,t} + \theta_{i,t}$$

$$\tilde{\sigma}_{n,i,t} = \frac{Q_t K_{i,t}}{N_{i,t}} \tilde{\sigma}_{R,t}$$

Financial constraint $\theta_{i,t} \in \Theta_{i,t}$:

- $\Theta_{i,t} = \{0\}$: agent cannot issue "equity" securities
- $\Theta_{i,t} = \{(\chi_t - 1)\frac{Q_t K_{i,t}}{N_{i,t}} \sigma_{R,t}, \chi_t \geq \underline{\chi}\}$: "skin-in-the-game" constraint
- $\Theta_{i,t} = \mathbb{R}^d$: unconstrained agent

Balance sheets and flows of funds



Models Nested

- Complete markets with long run risk
  - Bansal & Yaron (2004)
  - Hansen, Heaton & Li (2008)
- Complete markets with heterogeneous preferences
  - Longstaff & Wang (2012)
  - Garleanu & Panageas (2015)
- Complete markets for agg. risk with idiosyncratic shocks
  - Di Tella (2017)
- Incomplete market/limited participation models
  - Basak & Cuoco (1998)
  - Kogan & Makarov & Uppal (2007)
  - He & Krishnamurthy (2012)
- Incomplete market/capital misallocation models
  - Brunnermeier & Sannikov (2014, 2016)

Numerical Solution

User-friendly web application to solve models, downloadable at
`https://larspeterhansen.org/mfr-suite/`

Code implemented in C++, user interface via Jupyter Notebook

What the software does

1. Compute Markov equilibrium of the model
    a. "Outer loop" to solve single-agent HJBs iteratively
    b. "Inner loop" to solve for (i) capital allocation first order (elliptic) PDE and (ii) equity issuance policy algebraic equation iteratively
2. Compute stationary distribution via backward operator discretization
3. Compute unconditional moments of interest
4. Compute impulse response functions and term structure of risk prices (solutions to parabolic PDEs)

Part II

Binding Constraints and Risk Aversion Heterogeneity

Binding Constraints and Risk Aversion Heterogeneity

A simple example to warm up

Economic setting of focus

- Experts are the only producers ($a_h = -\infty$)
- Only agg. TFP shocks
- Agents with equal IES $1/\rho = 2$
- 50% minimum equity retention
- Unique state variable $W_t := N_{e,t} / \left(N_{e,t} + N_{h,t} \right)$

Compare

- Homogeneous risk aversion ($\gamma_e = \gamma_h = 3$) vs.
- Heterogeneous risk aversion ($\gamma_e = 3 < \gamma_h = 8$)

Binding Constraints and Risk Aversion Heterogeneity

Model illustration with MFM toolkit

Binding Constraints and Risk Aversion Heterogeneity

Assume $\gamma_e = \gamma_h$

Equity retention policy $\chi = \max\left(\underline{\chi}, w\right)$

Diffusion coefficient $\sigma_w = (\chi\kappa - w)\sigma_R = 0$ whenever $\chi > \underline{\chi}$

Consequence: in unitary IES case, financial constraint is

- always binding if $\delta_e = \delta_h$, $\lambda_d > 0$ and $\nu < \underline{\chi}$
- never binding if $\delta_e = \delta_h$, $\lambda_d > 0$ and $\nu > \underline{\chi}$
- always binding if $\delta_e > \delta_h$ and $\lambda_d = 0$

Theoretical Justification

To simplify, assume away idiosyncratic TFP shocks, and note that complementary slackness condition for $\chi_t \geq \underline{\chi}$ can be written

$$0 = \min\left(\chi - \underline{\chi}, \Delta_e\right) \qquad \Delta_e = \sigma_R \cdot \left[\pi_e - \pi_h\right] \qquad \pi_i = \gamma_i \sigma_{n_i} + (\gamma_i - 1)\sigma_{\xi,i}$$

Note $\sigma_{\hat{R}} := \sqrt{v}\sigma_k + \sigma_{\hat{x}}' \partial_{\hat{x}} \log q$

Use the identities

$$\sigma_R = \frac{\sigma_{\hat{R}}}{1 - (\beta_e - 1) w \partial_w \log q} \qquad \sigma_w = (\chi\kappa - w)\,\sigma_R \qquad \sigma_{\xi,i} = \sigma_x' \partial_x \xi_i$$

$$\sigma_{n_h} = \frac{1 - \chi\kappa}{1 - w}\sigma_R \qquad \sigma_{n_e} = \frac{\chi\kappa}{w}\sigma_R$$

Theoretical Justification

Complementary slackness $0 = \min\left(\chi - \underline{\chi}, \Delta_e\right)$

$\gamma_h = \gamma_e$: one can show that $\Delta_e \sim (\chi - w)$ when constraint not binding

Thus, homogeneous risk-aversion means $\chi = \max\left(\underline{\chi}, w\right)$

Intuition:

- if $\chi > \underline{\chi}$, experts face "locally" complete markets
- Portfolio choice σ_n solves $\max \mu_n - \frac{\gamma}{2}|\sigma_n|^2 + (1-\gamma)\left(\sigma_x \sigma_n\right) \cdot \partial_x \xi$
- Complete markets $\sigma_n = \frac{\pi}{\gamma} + \frac{1-\gamma}{\gamma}\sigma_x' \partial_x \xi$
- $\gamma_e = \gamma_h \Rightarrow$ identical portfolios when $\chi > \underline{\chi}$
- $\sigma_{w,t} = W_t(1 - W_t)\left(\sigma_{n_e,t} - \sigma_{n_h,t}\right) = 0$

Constraint always binding or never binding depends on sign of
$\mu_w\left(\underline{\chi}, \hat{x}\right) = \underline{\chi}\left(1 - \underline{\chi}\right)\left(c_h^*\left(\underline{\chi}, \hat{x}\right) - c_e^*\left(\underline{\chi}, \hat{x}\right)\right) + \lambda_d(\nu - \underline{\chi})$

Part III

Financial Frictions' Impact on Equilibrium Outcomes

Financial Frictions and Equilibrium Outcomes

Model comparison: complete markets vs. financial frictions

Economic setting of focus

- Experts are the only producers ($a_h = -\infty$)
- Only agg. TFP shocks
- Experts more risk-tolerant than households ($\gamma_e < \gamma_h$)
- Unique state variable $W_t := N_{e,t} / \left(N_{e,t} + N_{h,t}\right)$

Compare

- 50% minimum equity retention vs.
- No financial friction

Literature comparison

- Brunnermeier & Sannikov (2016) or He & Krishnamurthy (2012) with heterogeneous risk-aversion vs.
- Garleanu & Panageas (2015)

Financial Frictions and Equilibrium Outcomes

Model illustration with MFM toolkit

Part IV

Long Run Risk and Financial Frictions

Long Run Risk in Complete Market Models

Brief reminder of complete market result with unitary IES

Agent continuation value $\log U_t = \log K_t + \xi_t$

$$\xi_t = \beta_0 + \beta_{1z} Z_t + \beta_{1v} V_t$$
$$\pi_t = \sqrt{V_t}\left[(\gamma - 1)\left(\beta_{1z}\sigma_z + \beta_{1v}\sigma_v\right) + \gamma\sigma_k\right]$$

Coefficients β_{1z}, β_{1v} satisfy

- $\beta_{1z} = 1/(\lambda_z + \delta)$
- β_{1v} is the negative root to a quadratic equation

Model comparison: complete markets vs. financial frictions

Economic setting of focus

- Experts are the only producers ($a_h = -\infty$)
- agg. TFP shocks, growth rate and stochastic volatility shocks
- Identical preferences, $\gamma_i = 3$ and $\rho_i = 1$
- 3 state variables $X_t := (Z_t, V_t, W_t)$

Compare

- 50% minimum equity retention vs.
- No financial friction

Literature comparison

- Brunnermeier & Sannikov (2016) or He & Krishnamurthy (2012) with long run risk vs.
- Bansal & Yaron (2004)

Model illustration with MFM toolkit

Part V

Long Run Risk and Capital Misallocation

Economic setting of focus

- Experts and households can both produce ($a_e > a_h > -\infty$)
- No equity issuance allowed
- agg. TFP shocks and stochastic volatility shocks
- Identical preferences, $\gamma_i = 3$ and $\rho_i = 1$
- 2 state variables $X_t := (V_t, W_t)$

Question: how does stochastic volatility affect capital misallocation?

- 50% minimum equity retention vs.
- No financial friction

Model illustration with MFM toolkit

Summary

Large class of models that can be investigated with MFM toolkit

Robust numerical solution method that can handle multiple state variables

Preliminary model investigations suggest that

- Financial frictions interact in non-trivial ways with different types of shocks – in particular stochastic volatility shocks
- Preference heterogeneity can alter significantly the dynamic properties of the competitive equilibrium – from an environment with always-binding constraints to an environment with occasionally (and sometimes never!) binding constraints
- Environments with “skin-in-the-game” constraints might lead to low persistence of crisis regime compared to corresponding complete markets’ environments