Valuation Dynamics in Models with Financial Frictions

Lars Peter Hansen (University of Chicago)
Joseph Huang (University of Chicago)
Paymon Khorrami (University of Chicago)
Fabrice Tourre (Northwestern University)

UZH – Mini-Conference on Heterogeneous Agents Macroeconomics

July 4, 2018 – Happy USA Independence Day

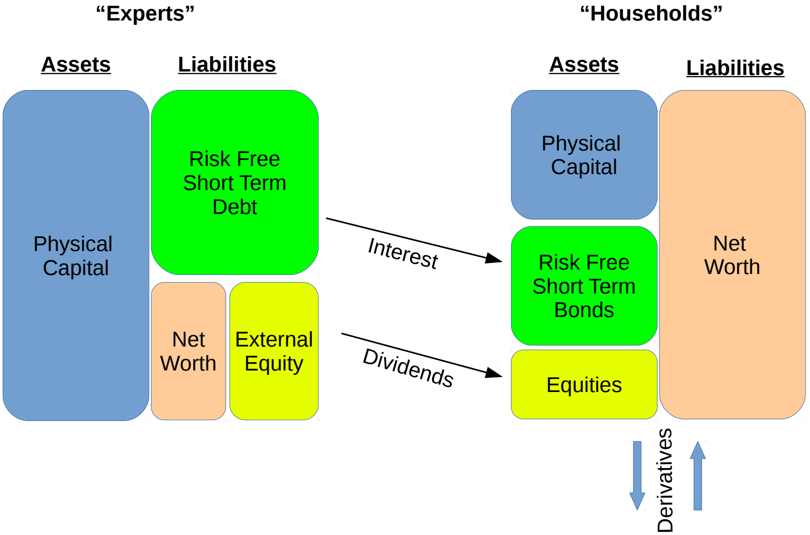
Research Objective

- **Research Goal**: Compare/contrast implications of macroeconomic models with financial frictions through study of their non-linear transmission mechanisms
- **Environment**
  - Continuous time with Brownian shocks
  - Two types of agents (one of them is like "financial intermediary")
  - Heterogeneous productivity, financial constraints, preferences
- **Comparison Targets**
  - Macroeconomic quantity implications
  - Asset pricing implications
  - Macro- and micro-prudential policies
- **Approach**: Nesting model

"Nesting" Model

- **Technology** Technology
  - A-K production function with $a_e \geq a_h$ and adjustment costs
  - TFP shocks (also called "capital quality shocks")
  - growth rate and stochastic vol shocks (long-run risk)
  - idiosyncratic shocks (nothing on this today)
- **Markets** Markets
  - Capital traded with shorting constraint
  - Experts face "skin-in-the-game" equity issuance constraint
- **Preferences** Preferences
  - Recursive utility, discount rate ρ, EIS ψ^{-1}, risk aversion γ
  - Households and experts potentially different
  - OLG for technical reasons

"Nesting" Model



Models Nested

- **Complete markets with long run risk**
  - Bansal & Yaron (2004)
  - Hansen, Heaton & Li (2008)
- **Complete markets with heterogeneous preferences**
  - Longstaff & Wang (2012)
  - Garleanu & Panageas (2015)
- **Incomplete market / limited participation**
  - Basak & Cuoco (1998)
  - Kogan & Makarov & Uppal (2007)
  - He & Krishnamurthy (2012)
- **Incomplete market / capital misallocation**
  - Brunnermeier & Sannikov (2014)
- **Complete markets for agg. risk with idiosyncratic shocks**
  - Di Tella (2017)

Overview of Solution Method

- **Markov equilibrium** – aggregate state vector X_t:
  - **Exogenous states** g_t (growth), s_t (agg. stochastic vol.), and ς_t (idio. stochastic vol.)
  - **Endogenous state** $w_t := \frac{N_{e,t}}{N_{e,t}+N_{h,t}}$ (wealth share)

Overview of Solution Method

- **Markov equilibrium** – aggregate state vector X_t:
  - **Exogenous states** g_t (growth), s_t (agg. stochastic vol.), and ς_t (idio. stochastic vol.)
  - **Endogenous state** $w_t := \frac{N_{e,t}}{N_{e,t}+N_{h,t}}$ (wealth share)

- "Value function" approach: $V_i\left(n_{i,t}, X_t\right) = n_{i,t}^{1-\gamma_i}\xi_i(X_t)$

Overview of Solution Method

- **Markov equilibrium** – aggregate state vector X_t:
  - **Exogenous states** g_t (growth), s_t (agg. stochastic vol.), and ς_t (idio. stochastic vol.)
  - **Endogenous state** $w_t := \frac{N_{e,t}}{N_{e,t}+N_{h,t}}$ (wealth share)

- "Value function" approach: $V_i\left(n_{i,t}, X_t\right) = n_{i,t}^{1-\gamma_i}\xi_i(X_t)$

- (ξ_e, ξ_h) solutions to second order non-linear PDEs – implicit FD scheme with artificial time derivative ("false transient")

Overview of Solution Method

- **Markov equilibrium** – aggregate state vector X_t:
  - **Exogenous states** g_t (growth), s_t (agg. stochastic vol.), and ς_t (idio. stochastic vol.)
  - **Endogenous state** $w_t := \frac{N_{e,t}}{N_{e,t}+N_{h,t}}$ (wealth share)
- "Value function" approach: $V_i\left(n_{i,t}, X_t\right) = n_{i,t}^{1-\gamma_i}\xi_i(X_t)$
- (ξ_e, ξ_h) solutions to second order non-linear PDEs – implicit FD scheme with artificial time derivative ("false transient")
- Each time-step: compute aggregate state dynamics and prices using the value functions from the previous time-step

Overview of Solution Method

- **Markov equilibrium** – aggregate state vector X_t:
  - **Exogenous states** g_t (growth), s_t (agg. stochastic vol.), and ς_t (idio. stochastic vol.)
  - **Endogenous state** $w_t := \frac{N_{e,t}}{N_{e,t}+N_{h,t}}$ (wealth share)
- "Value function" approach: $V_i\left(n_{i,t}, X_t\right) = n_{i,t}^{1-\gamma_i}\xi_i(X_t)$
- (ξ_e, ξ_h) solutions to second order non-linear PDEs – implicit FD scheme with artificial time derivative ("false transient")
- Each time-step: compute aggregate state dynamics and prices using the value functions from the previous time-step
- Endogenous state partition due to occasionally-binding constraints

Overview of Solution Method

- **Markov equilibrium** – aggregate state vector X_t:
  - **Exogenous states** g_t (growth), s_t (agg. stochastic vol.), and ς_t (idio. stochastic vol.)
  - **Endogenous state** $w_t := \frac{N_{e,t}}{N_{e,t}+N_{h,t}}$ (wealth share)
- "Value function" approach: $V_i\left(n_{i,t}, X_t\right) = n_{i,t}^{1-\gamma_i}\xi_i(X_t)$
- (ξ_e, ξ_h) solutions to second order non-linear PDEs – implicit FD scheme with artificial time derivative ("false transient")
- Each time-step: compute aggregate state dynamics and prices using the value functions from the previous time-step
- Endogenous state partition due to occasionally-binding constraints
- Implementation in C++ allowing for HPC

Numerical Implementation: Value Functions

Statement of the problem. Scaled value functions ξ_i solve PDEs like

$$0 = K_i + A_i\xi_i + B_i \cdot \partial_x \xi_i + \mathrm{trace}[C_i C_i' \partial_{xx'} \xi_i], \quad x = (w, g, s, \varsigma),$$

where the coefficients are:

$$\begin{aligned} K_i &= K_i(x, \xi_e, \xi_h, \partial_x \xi_e, \partial_x \xi_h) \\ A_i &= A_i(x, \xi_e, \xi_h, \partial_x \xi_e, \partial_x \xi_h) \\ B_i &= B_i(x, \xi_e, \xi_h, \partial_x \xi_e, \partial_x \xi_h) \\ C_i &= C_i(x, \xi_e, \xi_h, \partial_x \xi_e, \partial_x \xi_h) \end{aligned}$$

The dependence of A, B, C on (ξ_e, ξ_h) arises due to general equilibrium.

We solve this PDE system with a 2-step iterative approach:

- given coefficients, we solve the linear PDE and obtain $\{\xi_i\}_{i=e,h}$
- given PDE solution $\{\xi_i\}_{i=e,h}$, we update coefficients

Numerical Implementation: Value Functions

Step 1. Augment the PDE with a "false transient," which is an artificial time-derivative $\partial_t \xi_i$:

$$\partial_t \xi_i = K_i + A_i \xi_i + B_i \cdot \partial_x \xi_i + \text{trace}[C_i C_i' \partial_{xx'} \xi_i],$$

where

$$\begin{aligned} K_i &= K_i(x, \xi_e, \xi_h, \partial_x \xi_e, \partial_x \xi_h) \\ A_i &= A_i(x, \xi_e, \xi_h, \partial_x \xi_e, \partial_x \xi_h) \\ B_i &= B_i(x, \xi_e, \xi_h, \partial_x \xi_e, \partial_x \xi_h) \\ C_i &= C_i(x, \xi_e, \xi_h, \partial_x \xi_e, \partial_x \xi_h) \end{aligned}$$

Numerical Implementation: Value Functions

Step 2. Given an iterant or guess $(\xi_e^{(t)}, \xi_h^{(t)})$, we substitute the coefficients $(K_i^{(t)}, A_i^{(t)}, B_i^{(t)}, C_i^{(t)})$.

$$\partial_t \xi_i = K_i^{(t)} + A_i^{(t)} \xi_i + B_i^{(t)} \cdot \partial_x \xi_i + \text{trace}[C_i^{(t)} C_i^{(t)\prime} \partial_{xx'} \xi_i],$$

where

$$\begin{aligned}
K_i^{(t)} &= K_i(x, \xi_e^{(t)}, \xi_h^{(t)}, \partial_x \xi_e^{(t)}, \partial_x \xi_h^{(t)}) \\
A_i^{(t)} &= A_i(x, \xi_e^{(t)}, \xi_h^{(t)}, \partial_x \xi_e^{(t)}, \partial_x \xi_h^{(t)}) \\
B_i^{(t)} &= B_i(x, \xi_e^{(t)}, \xi_h^{(t)}, \partial_x \xi_e^{(t)}, \partial_x \xi_h^{(t)}) \\
C_i^{(t)} &= C_i(x, \xi_e^{(t)}, \xi_h^{(t)}, \partial_x \xi_e^{(t)}, \partial_x \xi_h^{(t)})
\end{aligned}$$

Numerical Implementation: Value Functions

Step 3. Discretize the time derivatives and write all spatial derivatives in terms of $\xi_i^{(t+\Delta)}$ ("implicit", as opposed to "explicit" scheme), i.e.,

$$\frac{\xi_i^{(t+\Delta)} - \xi_i^{(t)}}{\Delta} = K_i^{(t)} + A_i^{(t)} \xi_i^{(t+\Delta)} + B_i^{(t)} \cdot \partial_x \xi_i^{(t+\Delta)} + \mathrm{tr}[C_i^{(t)} C_i^{(t)\prime} \partial_{xx'} \xi_i^{(t+\Delta)}],$$

where

$$\begin{aligned}
K_i^{(t)} &= K_i(x, \xi_e^{(t)}, \xi_h^{(t)}, \partial_x \xi_e^{(t)}, \partial_x \xi_h^{(t)}) \\
A_i^{(t)} &= A_i(x, \xi_e^{(t)}, \xi_h^{(t)}, \partial_x \xi_e^{(t)}, \partial_x \xi_h^{(t)}) \\
B_i^{(t)} &= B_i(x, \xi_e^{(t)}, \xi_h^{(t)}, \partial_x \xi_e^{(t)}, \partial_x \xi_h^{(t)}) \\
C_i^{(t)} &= C_i(x, \xi_e^{(t)}, \xi_h^{(t)}, \partial_x \xi_e^{(t)}, \partial_x \xi_h^{(t)})
\end{aligned}$$

To insure scheme "monotonicity",

- "Upwinding" for discretization of $\partial_x \xi_i^{(t+\Delta)}$;
- Cross-partial derivatives computed using $\xi_i^{(t)}$ and added into $K_i^{(t)}$

Numerical Implementation: Value Functions

Step 4. By discretizing the spatial derivatives $\partial_x \xi_i^{(t+\Delta)}$ and $\partial_{xx'} \xi_i^{(t+\Delta)}$, the PDE becomes a system of linear equations in the unknown value function at the discretization points:

$$\left[I - \Delta L_i^{(t)}\right] \xi_i^{(t+\Delta)} = \xi_i^{(t)} + \Delta K_i^{(t)}$$

Solve this system for $(\xi_e^{(t+\Delta)}, \xi_h^{(t+\Delta)})$. Coded with assistance from Scheidegger (2011).

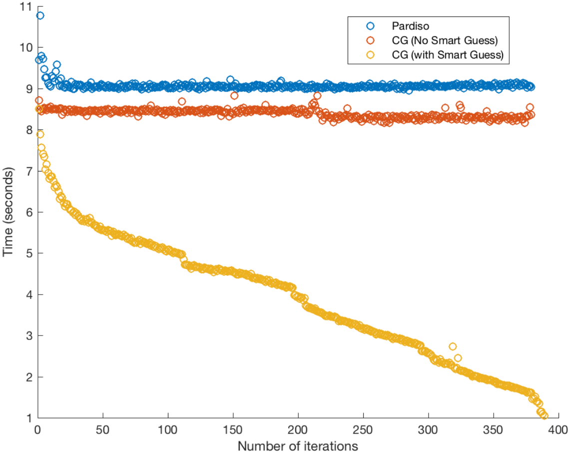
Numerical Implementation: Value Functions

Computational Considerations.

- Brownian information structure implies $L_i^{(t)}$ is a highly sparse matrix, with $I - \Delta L_i^{(t)}$ diagonally dominant for Δ sufficiently small
- Solving $\left[I - \Delta L_i^{(t)}\right] \xi_i^{(t+\Delta)} = \xi_i^{(t)} + \Delta K_i^{(t)}$:
  - direct approach: LU decomposition with PARDISO 6.0. See Kourounis, Fuchs, Schenk (2018); Verbosio, De Coninck, Kourounis, Schenk (2017); De Coninck, De Baets, Kourounis, Verbosio, Schenk, Maenhout, and Fostier (2016); `https://www.pardiso-project.org`.
  - iterative approach: conjugate gradient (CG) for symmetrized system, using different preconditioners and utilizing initial guess from previous time iteration.

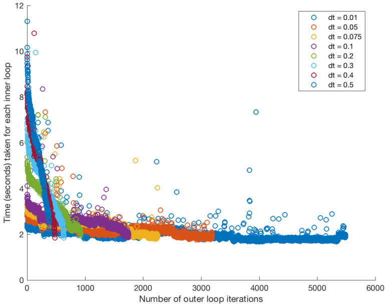
Numerical Implementation: Value Functions

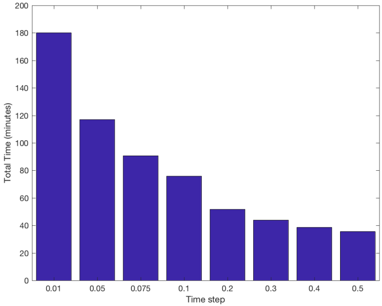
LU versus CG. Solve $\left[I - \Delta L_i^{(t)}\right] \xi_i^{(t+\Delta)} = \xi_i^{(t)} + \Delta K_i^{(t)}$ for $\xi_i^{(t+\Delta)}$



Numerical Implementation: Value Functions

Time-step trade-off with CG. Lower Δ means more iterations to converge, but better initial guesses in each iteration (and better matrix conditioning).





Numerical Implementation: Value Functions

Other computational issues.

- Explicit versus Implicit scheme
- Preconditioners for CG
- Non-uniform grids
- GPU computing suited to explicit scheme

Numerical Implementation: Constraints

Statement of the problem. Capital distribution $\kappa \in [0,1]$ and expert equity issuance $\chi \in [\underline{\chi}, 1]$ determine occasionally-binding constraints

$$0 = \min(1-\kappa, -\alpha_h)$$
$$0 = \min(\chi - \underline{\chi}, \alpha_e),$$

where α_i is agent i's endogenous premium on capital (relative to financial securities that replicate capital's shocks)

Economic intuition.

- Experts hold all capital ($\kappa = 1$) if and only if households obtain no premium for holding it ($\alpha_h < 0$)
- Experts issue as much equity as possible ($\chi = \underline{\chi}$) if and only if their inside equity premium exceeds the outside equity premium ($\alpha_e > 0$)

Numerical Implementation: Constraints

Variational inequalities. Algebraic equations on part of the state space (when constraints bind) and first-order non-linear elliptic PDEs on the complement (when constraints are slack).

$$0 = \min(1 - \kappa, -\alpha_h)$$
$$0 = \min(\chi - \underline{\chi}, \alpha_e),$$

where

$$\alpha_h = F_h(x, \kappa, \partial_x \kappa, \chi, \partial_x \chi)$$
$$\alpha_e = F_e(x, \kappa, \partial_x \kappa, \chi, \partial_x \chi).$$

Solution method.

- Explicit FD scheme with false transient and "CFL" condition, e.g.,

$$\frac{\kappa^{(\tau+\tilde{\Delta})} - \kappa^{(\tau)}}{\tilde{\Delta}} = \min\left(1 - \kappa^{(\tau)}, F_h(x, \kappa^{(\tau)}, \partial_x \kappa^{(\tau)}, \chi^{(\tau)}, \partial_x \chi^{(\tau)})\right)$$

- See Oberman (2006) for nonlinear first-order PDE schemes

Diagnostic Tools I

- **Quantities**
  - Consumption/wealth ratio $(c_i/n_i)(x)$
  - Investment rate $\iota(x)$
  - Output growth $\mu_y(x)$
- **Prices**
  - Risk-free rate $r(x)$
  - Risk-price vectors $\pi_i(x)$ (one per agent)
  - Capital price $q(x)$
- **State dynamics**
  - Drift $\mu_X(x)$ and diffusion $\sigma_X(x)$ of aggregate state vector
  - Ergodic density $f(x)$

Diagnostic Tools II

- Transition dynamics and valuation through **altering cashflow exposure to shocks**
- Focused on stochastically growing cashflows Y_t, C_t, $C_{e,t}$, $C_{h,t}$
- **Shock-exposure elasticities:** effect on future expected cashflow
- **Shock-cost elasticities:** effect on today's cashflow price
- **Shock-price elasticities:** effect on log expected returns
  - difference between shock-exposure and shock-cost elasticities
  - pricing counterpart to impulse response functions

Diagnostic Tools II

- Consider a martingale perturbation H_t^s in direction ν

$$
\begin{aligned}
d\log H_t^s &= -\frac{\|\nu(X_t)\|^2}{2}dt + \nu(X_t)\cdot dZ_t \qquad 0 \le t \le s \\
d\log M_t &= \mu_M(X_t)dt + \sigma_M(X_t)\cdot dZ_t \\
\epsilon_M(x,t) &:= \lim_{s\to 0}\frac{1}{s}\log\mathbb{E}\left[\frac{M_t}{M_0}H_t^s | X_0 = x\right]
\end{aligned}
$$

Diagnostic Tools II

- Consider a martingale perturbation H_t^s in direction ν

$$
\begin{aligned}
d \log H_t^s &= -\frac{\|\nu(X_t)\|^2}{2} dt + \nu(X_t) \cdot dZ_t \qquad 0 \le t \le s \\
d \log M_t &= \mu_M(X_t) dt + \sigma_M(X_t) \cdot dZ_t \\
\epsilon_M(x,t) &:= \lim_{s \to 0} \frac{1}{s} \log \mathbb{E}\left[\frac{M_t}{M_0} H_t^s | X_0 = x \right]
\end{aligned}
$$

- Applications for a cash-flow C_t received at time t
  - Shock-exposure elasticity $\epsilon_C(x,t)$;
  - Shock-cost elasticity $\epsilon_{SC}(x,t)$;
  - Shock-price elasticity $\epsilon_C(x,t) - \epsilon_{SC}(x,t)$

Diagnostic Tools II

- Consider a martingale perturbation H_t^s in direction ν

$$d \log H_t^s = -\frac{\|\nu(X_t)\|^2}{2} dt + \nu(X_t) \cdot dZ_t \qquad 0 \le t \le s$$

$$d \log M_t = \mu_M(X_t) dt + \sigma_M(X_t) \cdot dZ_t$$

$$\epsilon_M(x, t) := \lim_{s \to 0} \frac{1}{s} \log \mathbb{E}\left[\frac{M_t}{M_0} H_t^s | X_0 = x\right]$$

- Applications for a cash-flow C_t received at time t
  - Shock-exposure elasticity $\epsilon_C(x, t)$;
  - Shock-cost elasticity $\epsilon_{SC}(x, t)$;
  - Shock-price elasticity $\epsilon_C(x, t) - \epsilon_{SC}(x, t)$

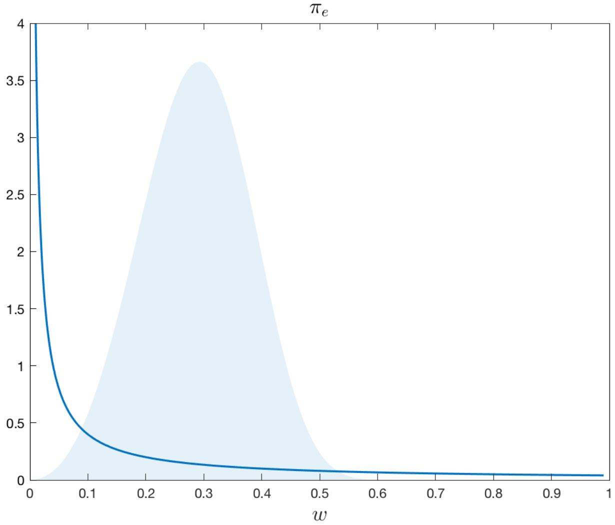
- Two interpretations
  - Altering the probability distribution of cashflow
  - Altering the exposure of cashflow (Malliavin derivative)
  - See Borovička-Hansen-Scheinkman (2014, *Math and Fin Econ*) for equivalence to nonlinear IRFs under Brownian shocks

Baseline Model: 1D limited participation model

Baseline version of model is like Basak-Cuoco (1998)

- Experts are the only producers (i.e. $a_h = -\infty$)
- Skin-in-the-game constraint $\chi \equiv \underline{\chi} = 1$
- TFP shocks only
- log utility RRA $\gamma = 1$, EIS $\psi^{-1} = 1$

Baseline Model: 1D limited participation model



Always vs. Occasionally Binding Constraints

- When is an always-binding-constraint assumption legitimate?

Always vs. Occasionally Binding Constraints

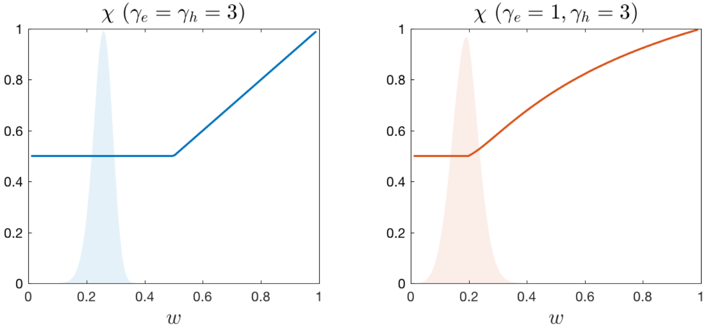
- When is an always-binding-constraint assumption legitimate?
- Economic setting of focus
  - Experts are the only producers (i.e. $a_h = -\infty$)
  - Skin-in-the-game constraint $\chi \geq \underline{\chi}$
  - TFP shocks only
  - EIS $\psi^{-1} = 1$

Always vs. Occasionally Binding Constraints

- When is an always-binding-constraint assumption legitimate?
- Economic setting of focus
  - Experts are the only producers (i.e. $a_h = -\infty$)
  - Skin-in-the-game constraint $\chi \geq \underline{\chi}$
  - TFP shocks only
  - EIS $\psi^{-1} = 1$
- Compare
  1. homogeneous RRA ($\gamma_e = \gamma_h$) vs.
  2. heterogeneous RRA ($\gamma_e < \gamma_h$)

Always vs. Occasionally Binding Constraints

Expert's risk-retention χ in the two models.



Always vs. Occasionally Binding Constraints

Proposition. If $\gamma_e = \gamma_h$ and w_t is the only state variable (i.e., 1-dimensional model), then $\Pr\{\exists t : \chi_t > \underline{\chi}\} = 0$.

Numerical result so far. Proposition above holds even in higher dimensions (i.e., (g_t, s_t) are state variables), as long as $\psi_e = \psi_h$ (same EIS).

Other Shocks and Financial Frictions

- How do financial frictions affect agents' attitudes about shocks?

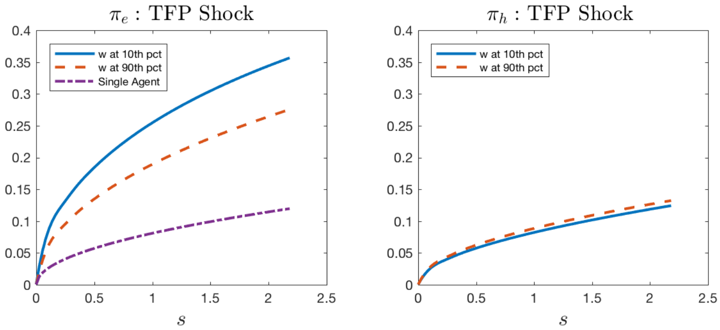
Other Shocks and Financial Frictions

- How do financial frictions affect agents' attitudes about shocks?
- Economic setting of focus
  - Experts are the only producers (i.e. $a_h = -\infty$)
  - Skin-in-the-game constraint $\chi \geq \underline{\chi}$
  - Shocks to TFP level, growth rate, and volatility
  - RRA $\gamma = 3$, EIS $\psi^{-1} = 1$

Other Shocks and Financial Frictions

- How do financial frictions affect agents' attitudes about shocks?
- Economic setting of focus
  - Experts are the only producers (i.e. $a_h = -\infty$)
  - Skin-in-the-game constraint $\chi \geq \underline{\chi}$
  - Shocks to TFP level, growth rate, and volatility
  - RRA $\gamma = 3$, EIS $\psi^{-1} = 1$
- Compare
  1. model with frictions ($\underline{\chi} = 0.5$) vs.
  2. model without frictions ($\underline{\chi} = 0$)

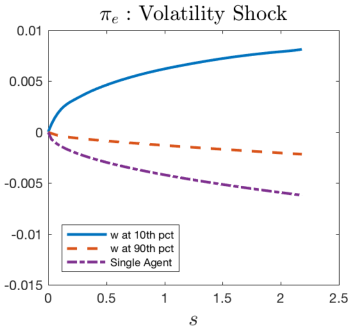
Expert's and Household's TFP risk prices $\pi_e^{(1)}, \pi_h^{(1)}$, along with the "Single Agent" TFP risk price.



Note: "Single Agent" denotes the "frictionless" model ($\underline{\chi} = 0$).

Other Shocks and Financial Frictions

Expert's and Household's volatility risk prices $\pi_e^{(3)}, \pi_h^{(3)}$, along with the "Single Agent" volatility risk price.



Note: "Single Agent" denotes the "frictionless" model ($\underline{\chi} = 0$).

Volatility Paradox

- How does volatility affect the wealth distribution under financial frictions? (c.f., Brunnermeier-Sannikov “volatility paradox”)

Volatility Paradox

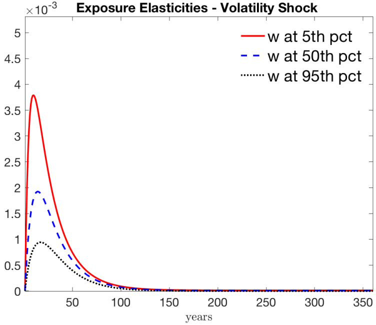
- How does volatility affect the wealth distribution under financial frictions? (c.f., Brunnermeier-Sannikov "volatility paradox")
- Economic setting of focus
  - Experts are the only producers (i.e. $a_h = -\infty$)
  - Skin-in-the-game constraint $\chi \geq \underline{\chi}$
  - Shocks to TFP level, growth rate, and volatility
  - EIS $\psi^{-1} = 1$

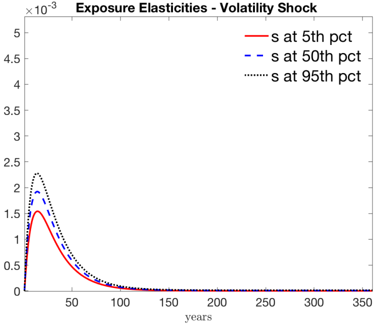
Volatility Paradox

- How does volatility affect the wealth distribution under financial frictions? (c.f., Brunnermeier-Sannikov "volatility paradox")
- Economic setting of focus
  - Experts are the only producers (i.e. $a_h = -\infty$)
  - Skin-in-the-game constraint $\chi \geq \underline{\chi}$
  - Shocks to TFP level, growth rate, and volatility
  - EIS $\psi^{-1} = 1$
- Compare the dynamic and long-run effects of s_t on w_t
  1. diffusion of volatility σ_s
  2. mean-reversion of volatility shocks λ_s
  3. skin-in-the-game $\underline{\chi}$
  4. common risk aversion γ

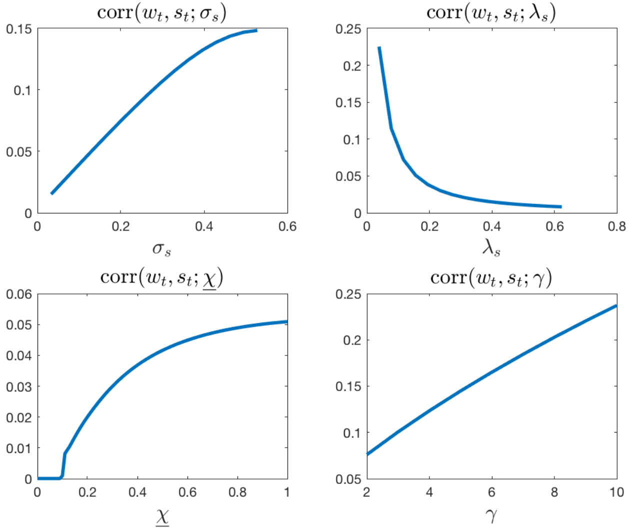
Volatility Paradox

Volatility shock-exposure elasticity of w_t.





Corr(w_t, s_t) as a function of model parameters.



Productivity vs. Risk-Tolerance

- Are the "expert" agents in the economy more productive or more risk-tolerant?

Productivity vs. Risk-Tolerance

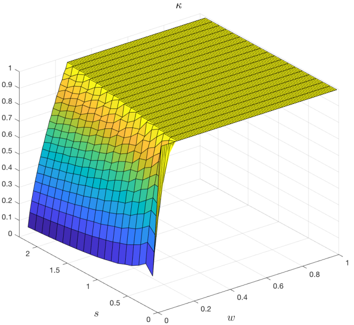
- Are the "expert" agents in the economy more productive or more risk-tolerant?
- Economic setting of focus
  - Experts and households can both produce (i.e. $a_e \geq a_h > -\infty$)
  - No equity-issuance $\chi \equiv \underline{\chi} = 1$
  - Shocks to TFP level, growth rate, and volatility
  - EIS $\psi^{-1} = 1$

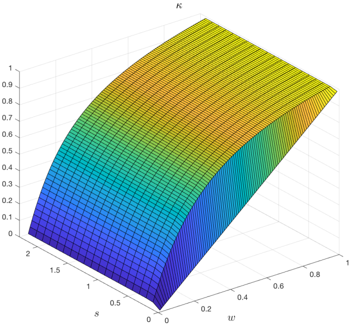
Productivity vs. Risk-Tolerance

- Are the "expert" agents in the economy more productive or more risk-tolerant?
- Economic setting of focus
  - Experts and households can both produce (i.e. $a_e \geq a_h > -\infty$)
  - No equity-issuance $\chi \equiv \underline{\chi} = 1$
  - Shocks to TFP level, growth rate, and volatility
  - EIS $\psi^{-1} = 1$
- Compare
  1. experts more productive ($a_e > a_h$ but $\gamma_e = \gamma_h$) vs.
  2. experts more risk-tolerant ($\gamma_e < \gamma_h$ but $a_e = a_h$)

Productivity vs. Risk-Tolerance

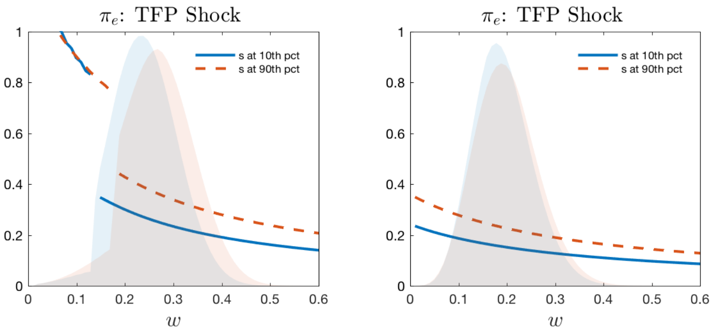
Capital distribution κ as a function of (w, s) in the two models.





Productivity vs. Risk-Tolerance

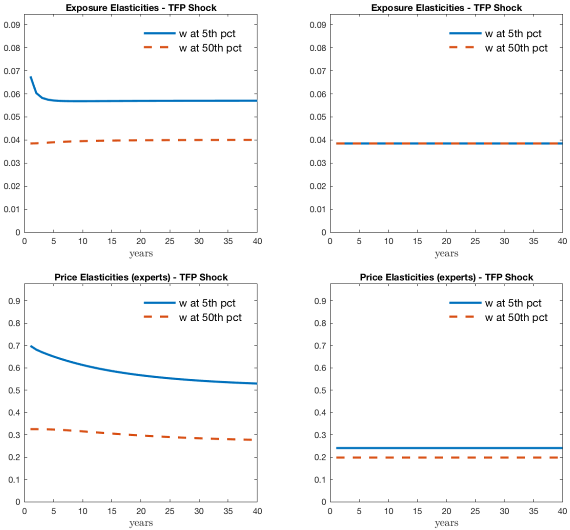
Expert's TFP risk price $\pi_e^{(1)}$ in the two models.



Note: adjusted other parameters to make the wealth distributions more similar across the two models.

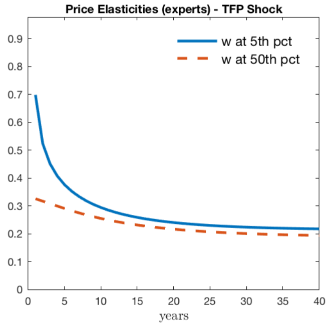
Productivity vs. Risk-Tolerance

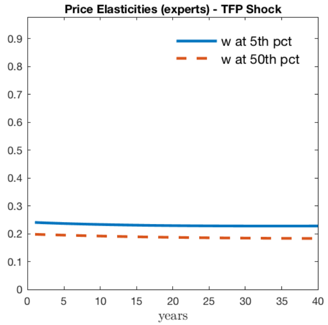
TFP shock-exposure and -price elasticities of C_t in both models.



Productivity vs. Risk-Tolerance

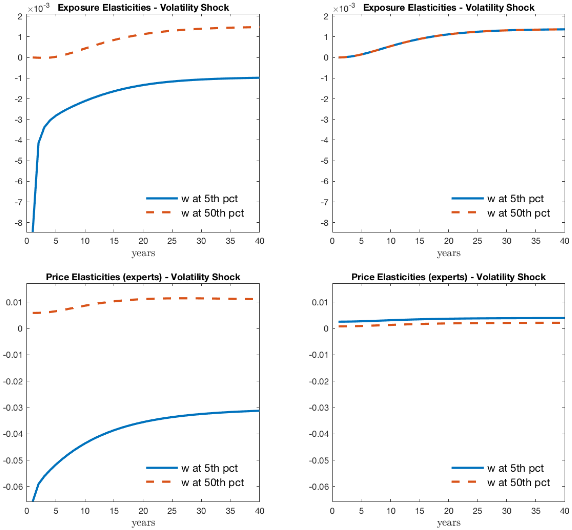
TFP shock-exposure elasticity of $C_{e,t}$ in both models.





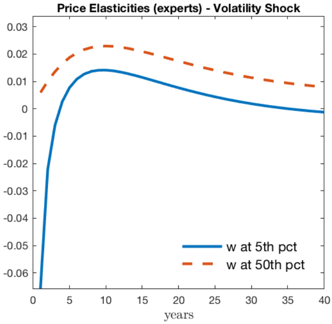
Productivity vs. Risk-Tolerance

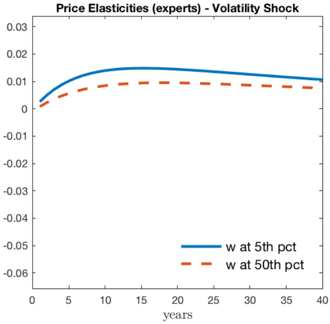
Volatility shock-exposure and -price elasticities of C_t in both models.



Productivity vs. Risk-Tolerance

Volatility shock-exposure elasticity of $C_{e,t}$ in both models.





Conclusion / Next Steps

- Additional computational explorations (e.g., GPU computing)
- Additional types of financial constraints (e.g., leverage constraints)
- User-friendly web application to compare and contrast models...
  `https://modelcomparisons.shinyapps.io/modelcomparisonssite/`

Technology

- Efficiency units of capital k_t follow

$$dk_t = k_t \left[(g_t + \iota_t - \delta)\, dt + \sqrt{s_t}\sigma \cdot dZ_t \right], \tag{1}$$

- Exogenous state variables (s_t, g_t) follow

$$dg_t = \lambda_g(\overline{g} - g_t)dt + \sqrt{s_t}\sigma_g \cdot dZ_t \tag{2}$$

$$ds_t = \lambda_s(\overline{s} - s_t)dt + \sqrt{s_t}\sigma_s \cdot dZ_t \tag{3}$$

- Adjustment costs: investment $\iota_t k_t dt$ costs $\Phi(\iota_t)\, k_t dt$ in output

back

Markets

- Capital is freely traded (subject to no-shorting constraints), at price q_t

$$dq_t = q_t[\mu_{q,t} dt + \sigma_{q,t} \cdot dZ_t] \tag{4}$$

- Households facing dynamically complete markets, leading to SDF

$$dS_{h,t} = -S_{h,t}[r_t dt + \pi_{h,t} \cdot dZ_t] \tag{5}$$

- Experts face skin-in-the-game constraint via minimum risk retention:

$$\chi_t \geq \underline{\chi} \tag{6}$$

- Experts' SDF differs from Households' SDF:

$$dS_{e,t} = -S_{e,t}[r_t dt + \pi_{e,t} \cdot dZ_t] \tag{7}$$

back

Preferences and Single-Agent Problem

- Agent i will solve the following problem:

$$
\begin{aligned}
U_{i,t} &= \max_{\{k_i \geq 0, c_i, \theta_i, \iota_i\}} \mathbb{E}\left[\int_t^{+\infty} \varphi\left(c_{i,s}, U_{i,s}\right) ds\right] \\
\text{s.t. } \frac{dn_{i,t}}{n_{i,t}} &= \left[\mu_{n,i,t} - \frac{c_{i,t}}{n_{i,t}}\right] dt + \sigma_{n,i,t} \cdot dZ_t \\
\mu_{n,i,t} &= r_t + \frac{q_t k_{i,t}}{n_{i,t}} \left(\mu_{R,i,t} - r_t\right) + \theta_{i,t} \cdot \pi_t \\
\sigma_{n,i,t} &= \frac{q_t k_{i,t}}{n_{i,t}} \sigma_{R,t} + \theta_{i,t} \\
\theta_{i,t} &\in \Theta_{i,t}
\end{aligned}
$$

- Financial constraint set $\Theta_{i,t}$:
  - $\Theta_{i,t} = \{0\}$: agent cannot issue "equity" securities
  - $\Theta_{i,t} = \{(\chi_t - 1)\frac{q_t k_{i,t}}{n_{i,t}} \sigma_{R,t}, \chi_t \geq \underline{\chi}\}$: "skin-in-the-game" constraint
  - $\Theta_{i,t} = \mathbb{R}^d$: unconstrained agent

back