Human Capital in a Time of Low Interest Rates*
[Preliminary and Incomplete]

Paymon Khorrami
Duke Fuqua
paymon.khorrami@duke.edu

Jung Sakong
Federal Reserve Bank of Chicago
jung.sakong@frbchi.org

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Abstract

We argue that a long-term low interest rate environment can cause labor income inequality, the emergence of the working rich, and reduced intergenerational mobility. We provide a simple model with endogenous human capital accumulation and credit constraints to demonstrate this causal link. The mechanism operates through a tilting of the human capital gradient: wealthy households, moreso than poor households, will increase human capital investment in response to low rates. Normatively, these tilting responses to low rates are inefficient, but higher capital taxes is not an ideal response. We find empirical support for our tilting mechanism over the last 60 years in the US. Quantitatively, we show that the endogenous human capital investment response to low interest rates can account for a 17% rise in cross-sectional labor income variance (higher inequality) and a 7% higher parent-child labor income inter-generational elasticity (lower mobility).

JEL Codes: D30, E21, E22, E24, E25

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> The professional classes especially, while generally eager to save some capital *for* their children, are even more on the alert for opportunities of investing it *in* them.
>
> —Alfred Marshall, *Principles of Economics*, 1890

How is the labor income distribution affected by a low interest rate environment? Over the past 40-50 years, interest rates in advanced economies have been declining and are projected to remain low (Laubach and Williams, 2003; Holston et al., 2017). Over a similar timeframe, and within similar advanced economies, labor income dispersion has risen (Piketty and Saez, 2003, 2006; Piketty et al., 2018). This paper offers theory and evidence on the effect of low interest rates on high labor income inequality, via uneven human capital accumulation.

Certainly, interest rate and inequality dynamics involve a plethora of economic, technological, cultural, and regulatory factors, making it difficult to identify any particular connection between them. The decline in interest rates has been linked to, among other things, the fall in aggregate growth rates (Laubach and Williams, 2003); the "global savings glut" (Bernanke, 2005); increased savings by the rich (Mian et al., 2020); demographic shifts (Rachel and Smith, 2017); or even monetary policy (Bianchi et al., 2022). On the other hand, the rise in labor income inequality is often attributed to rising skill premia, which can result from skill-biased technical change (Katz and Murphy, 1992; Acemoglu, 2002); declining equipment prices (Krusell et al., 2000); or other institutional features of the educational and regulatory environment (Goldin and Katz, 2010). Given the complexity and richness of the forces governing rates and inequality, our paper aims to connect the two phenomena by zooming into a relatively neoclassical, but underexplored, mechanism.

Like any real investment activity, human capital investment should rise as interest rates fall. Intuitively, the "discount rate" on human capital declines with interest rates, while its "cash flows" are future wages that are insensitive to interest rates. By contrast, low interest rates induce a fall in both the discount rate and future cash flows associated to financial assets. The net effect is an increase in the value of human capital, relative to financial assets, and thus a greater incentive to invest in education. With credit constraints, investment rises less for poorer households, because they are reluctant to borrow. Said differently, poorer households' effective discount rate also includes the shadow value of binding financial constraints, which is relatively insensitive to the interest rate. The result is a disproportionately large increase in educational investment for wealthier households, a phenomenon we refer to as *human capital tilting*. Human capital tilting in response to low rates is not an artifact of unusual assumptions that differ from the literature. In particular, our model studies a relatively standard family with neoclassical preferences consuming and investing in both financial and human capital, subject to borrowing constraints.

We find some empirical support for tilting: the rich-poor gap in parent-on-child educational investment has grown since 1980 in the US, a period of falling real interest rates. In the 20 years prior to 1980, interest rates fell and then rose, and the rich-poor educational investment gap mirrored this pattern, growing in the 1960s and falling in the 1970s. Unfortunately, data limitations prevent us from studying human capital tilting further back in time; the cross-section of educational expenditure is only observable since the 1960s.

Human capital tilting is the key driving mechanism behind a series of other theoretical results. In our model, falling interest rates cause greater labor income dispersion (Piketty et al., 2018; Blanchet et al., 2021), the emergence of the working rich (Piketty and Saez, 2003; Kaplan and Rauh, 2013; Smith et al., 2019; Eisfeldt et al., 2021), and lower intergenerational income mobility (Corak, 2013; Chetty et al., 2014; Davis and Mazumder, 2017; Chetty et al., 2017). Quantitatively, in response to 3% lower real rates, our baseline calibration generates a 17% rise in cross-sectional log labor income variance and a 7% higher parent-child labor income inter-generational elasticity.

We show how, from a normative perspective, human capital tilting and the associated changes in inequality and mobility are inefficient because human capital investment becomes less aligned with talent and more aligned with wealth. We also argue that using capital taxes to combat inequality is partially self-defeating, since capital taxes reduce financial returns, thereby amplifying human capital tilting and offsetting some of the inequality reduction.

We consider two alternative explanations for the changes in income distribution and dynamics—(i) skill premia and (ii) wealth inequality—both of which were also trending upwards in the last 40 years. A rising skill premium incentivizes educational investment, moreso for richer, less credit-constrained households. To distinguish our story from one based on the skill premium, notice that a decline in interest rates should expand investment, moreso for richer households, in *any* non-financial asset, such as housing. By contrast, a skill premium theory implies substitution away from housing. Rising wealth inequality can also generate human capital tilting, because credit constraints induce a monotonic dependence of educational investment on wealth. However, since poor households' decisions are most sensitive to financial wealth, a rise in wealth inequality generates tilting primarily through a reduction in educational investment by the poorest families, in contrast to the observed changes in the data.

Although the mechanisms we espouse are relatively neoclassical, our paper is unusual in connecting discount rates to labor income distributions. By and large, the literature on wage inequality has focused on forces that directly affect wages, e.g., skill-biased technical change (Katz and Murphy, 1992; Katz and Autor, 1999; Krusell et al., 2000; Acemoglu, 2002; Autor et al., 2003). In that context, Murphy and Topel (2016) show how human capital accumulation and inequality respond to changes in the skill premium. A recent literature on educational attainment emphasizes the importance of credit constraints (Lochner and Monge-Naranjo, 2011; Hai and Heckman, 2017; Caucutt and Lochner, 2020) but does not study the resulting income distribution nor the role of interest rates.

On the other hand, interest rates and other rates of return lie at the core of a fast-growing literature on wealth inequality (Saez and Zucman, 2016; Benhabib and Bisin, 2018; Benhabib et al., 2019; Fagereng et al., 2020, 2021; Greenwald et al., 2021; Hubmer et al., 2021). Universally, this literature has abstracted from the endogenous determination of labor income and skills.[1] For example, the recent study by Greenwald et al. (2021) argues that human wealth increases as interest rates fall, moreso for the poor and young, which tends to mitigate the effects of rising financial wealth inequality. The opposite conclusion holds once you allow for human capital investments under low interest rates: the presence of human capital amplifies financial inequalities.

Many theoretical studies of inequality zoom in on the extreme right tail (Jones, 2015; Gabaix et al., 2016; Gomez, 2017; Jones and Kim, 2018), partly because the tail has changed the most and partly because the economic mechanisms plausibly differ from those affecting the rest of the distribution. By contrast, our human capital forces are more applicable to the entire distribution (and perhaps least applicable to the tail).

A recent literature studies how monetary policy—and interest rates more generally—affects investment across the distribution of firms under financial frictions. This literature presents somewhat disparate results depending on the setting and details. For example, Ottonello and Winberry (2020) argue empirically that more financially-constrained firms are less sensitive to interest rates, whereas Jeenas (2019) argues the opposite using similar data but a different empirical specification. Gopinath et al. (2017) argues that both productivity and financial constraints matter for the heterogeneity of investment responses, and Asriyan et al. (2021) argues that general equilibrium forces can reverse

[1] The abstraction from labor incomes can be traced to sharp theoretical results proving that the statistical properties of top incomes are solely determined by those of capital income (Benhabib et al., 2011). Similarly, the statistical properties of top wealth shares are thought to be independent of the labor income process.

partial equilibrium responses in this context. Unlike these papers, we are focused on human capital investments and household income inequality, rather than firms.[2]

Finally, Auclert and Rognlie (2020) and Mian et al. (2021) have argued theoretically that income inequality can depress interest rates, through demand for savings. Together with our paper—which establishes the reverse causality—this suggests a potential two-way feedback between inequality and interest rates that may be worthy of investigation.

The remainder of the paper is organized as follows. Section 1 studies a simple one-period model to develop all of the theoretical insights. Section 2 provides empirical support to our main mechanisms. Section 3 quantifies the size of the model effects. Section 4 contains concluding remarks. Appendix A contains proofs; Appendix B explores the determinants of the investment-to-consumption ratio gradient; Appendix C explains which inequality measures are *not* addressed by our model; and Appendix D considers some model extensions.

1 Model

The following small, open economy is composed of parents and children. The population of parents begins with financial assets a, heritable skill z, and accumulated skills h, drawn from distribution F, i.e., $(a, z, h) \sim F$. Parents all have utility $u(\cdot)$ over their own consumption and decide how much to consume c, how much to invest x in the child's human capital, and how much wealth to bequeath a'. Given their budget constraint, $a' = R[a + y - c - x]$, where $y := \varepsilon z h$ is parental labor income, and $\varepsilon \sim \Xi$ is an idiosyncratic shock with mean 1. For simplicity, the shock ε is known by the parent at the time of decision-making (the only purpose of ε is to enrich income distributions in quantitative exercises). Parents additionally face the borrowing constraint $a + y - c - x \geq -b$, where $b \geq 0$. Finally, parents place utility weight β on their children.

Children inherit wealth a' and deterministic skill $z' := z^{\psi} \bar{z}^{1-\psi}$, with $\psi > 0$ capturing some amount of talent persistence (one can think of $\bar{z}$ as the average skill in the population of parents). From their inherited skill and the parental investment, children earn labor income $y' = \varepsilon' z' g(x; h)$, with idiosyncratic shock $\varepsilon' \sim \Xi$. Ultimately, children make no decisions and consume $c' := a' + y'$, after which the economy terminates. For simplicity, we assume parents evaluate children's utility as $u(\mathbb{E}[c'])$, i.e., the utility of

[2]Two other related papers, which may have consequences for inequality, are Liu et al. (2021) and Gomez and Gouin-Bonenfant (2020). Like us, both papers study how low interest rates amplify investment incentives (into market power and entrepreneurship, respectively) moreso for one subgroup (industry leaders and wealthy business-owners, respectively). Unlike these papers, we focus on human capital investment and the role of credit constraints (these papers study frictionless mechanisms).

expected consumption.[3] Thus, parents solve

$$\max_{c,x} u(c) + \beta u(\mathbb{E}[c'])$$
$$\text{subject to} \quad a + y - c - x \geq -b$$
$$\text{and} \quad \mathbb{E}[c'] := R[a + y - c - x] + z' g(x; h)$$

We assume the utility function and human capital production function are given by

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$
$$g(x; h) = h^{\zeta} x^{\alpha}, \quad \alpha + \zeta < 1.$$

These commonly-adopted functional forms have two properties which are convenient. Both satisfy an Inada condition at 0, which delivers interior optima, and both feature constant elasticity and curvature, so that none of our results emanate from exogenous cross-sectional heterogeneity in consumption or investment propensities. The explicit role for parental human capital h on the child's human capital formation, i.e., the function g, reflects the complementarities discussed in Cunha and Heckman (2007) and Becker et al. (2018). This feature is not critical for our theoretical results but helps our model quantitatively—going forward we will suppress the dependence of g on h for streamlined notation.

In our model, we will often want to clarify the mechanics by simplifying to the special case with no borrowing ($b = 0$) and log utility ($\gamma = 1$). The economics will be particularly clear in this stark example. Since we refer to this case so often, we give it the following name:

Definition 1. The *No-Borrowing-Log (NBL)* economy is defined by $b = 0$ and $\gamma = 1$.

1.1 "Tilting" mechanism

Optimality of consumption and investment imply

$$u'(c) = \beta R u'(\mathbb{E}[c']) + \lambda$$
$$\beta z' g'(x) u'(\mathbb{E}[c']) = \beta R u'(\mathbb{E}[c']) + \lambda,$$

[3]The fact that the parents care about a utility of *expected* child consumption is for simplicity (this shortcut is used in Becker et al. (2018) as well) and could alternatively be justified as a first-order approximation to an expected utility model for small shocks ε'. Consequently, parents will not make any precautionary decisions due to the child's income risk, which will allow for clean analytical results.

where λ is the Lagrange multiplier on the borrowing constraint. The consumption-savings decision is intertemporal, trading off parental utility against child utility. The investment decision, by contrast, is a static trade-off between financial returns and the returns to human capital, both in terms of the child's utility. Manipulating the investment FOC illuminates the economics; let us divide by $\beta u'(\mathbb{E}[c'])$ to obtain

$$z'g'(x) = R + \frac{\lambda}{\beta u'(\mathbb{E}[c'])}. \tag{1}$$

This says that the marginal benefit of human capital investment is optimally linked to an individual's "discount rate," where the discount rate not only includes the exogenous rate of return R but also the shadow cost of borrowing at the constraint $\frac{\lambda}{\beta u'(\mathbb{E}[c'])}$, which is denominated in units of child marginal utility. Consequently, a constrained parent has an effectively higher discount rate they apply to investment decisions.

Using the functional forms for u and g, unconstrained parents (with $\lambda = 0$) choose

$$x^* := \left(\frac{\alpha h^\zeta z'}{R}\right)^{\frac{1}{1-\alpha}} \tag{2}$$

$$c^* := \frac{R(a + y - x^*) + z'g(x^*)}{R + (\beta R)^{1/\gamma}}. \tag{3}$$

Define the set of borrowing-constrained parents (which is completely defined in terms of model parameters),

$$\mathcal{B} := \Big\{(a, z, h, \varepsilon) : c^* + x^* \geq b + a + \varepsilon z h\Big\}; \tag{4}$$

its complement $\mathcal{U} := \mathcal{B}^{\mathrm{c}}$ represents the unconstrained parents. Intuitively, borrowing-constrained parents should be those with low a, z, h, and ε, though the effects of z and h are technically ambiguous, since they all raise parental income but also the productivity of investment in children.

For constrained parents, we may combine the FOCs to eliminate λ. We obtain a condition equating the marginal utility of consumption to the marginal value of human capital investment:

$$\beta z'g'(x) = \frac{u'(c)}{u'(\mathbb{E}[c'])}. \tag{5}$$

Using the functional forms and the borrowing constraint, we have a single equation that determines investment x for constrained parents:

$$x_b = \left\{ x : (\alpha\beta z' h^{\zeta})^{\frac{1}{\gamma}} (a + b + y - x) = z' h^{\zeta} x^{\frac{1-\alpha+\alpha\gamma}{\gamma}} - Rbx^{\frac{1-\alpha}{\gamma}} \right\}. \tag{6}$$

Once x_b is determined from (6), consumption $c_b = a + b + y - x_b$ is obtained from the borrowing constraint. One can show that x_b is determined uniquely.

We start with the NBL economy of Definition 1, as a special case, to understand the key forces. The NBL economy has a closed-form solution, given by

Lemma 1. *If $b = 0$ and $\gamma = 1$, then*

$$x^* = \left(\frac{\alpha z' h^{\zeta}}{R} \right)^{\frac{1}{1-\alpha}} \quad \textit{and} \quad x_b = \frac{\alpha\beta}{1+\alpha\beta}(a+y)$$
$$c^* = \frac{a+y}{1+\beta} + \frac{z' g(x^*) - Rx^*}{R(1+\beta)} \quad \textit{and} \quad c_b = \frac{1}{1+\alpha\beta}(a+y).$$

From Lemma 1, we note that the primary effect of a change in returns R is in unconstrained human capital investment x^*. The effect of R on x^* is a pure portfolio choice effect: lower R decreases the value of financial investment relative to human investment, so x^* unambiguously rises. The effect of R on c^* is a wealth effect: lower R increases the net-present-value of child income through the term $\frac{z' g(x^*) - Rx^*}{R(1+\beta)}$, so c^* also unambiguously rises. In passing, notice how the presence of real assets creates an upward-bias for traditional inference about the EIS (i.e., despite unitary EIS, human capital implies an inverse relationship between c and R). Constrained parents' behavior is particularly simple: they consume and invest constant fractions of liquid net wealth $a + y$. A reduction in R has no effect because constrained parents cannot borrow more than they already do. There are also no balance-sheet effects here because of $b = 0$; indeed, constrained parents plan to leave their children with exactly zero financial bequests ($R[a + y - c_b - x_b] = 0$), so changes in R do not impact this plan. Although this is a stark effect, $b = 0$ may actually be a reasonable approximation, given the empirically small fraction of individuals with negative liquid assets.

The conventional economics of borrowing constraints—that financial assets matter for constrained folks—also apply here in a very transparent way. Higher assets a allow an increase in both consumption and investment c_b and x_b. Eventually, if a rises enough, x_b meets the unconstrained optimum x^*. By contrast, assets a have no effect on unconstrained investment x^*. The reason is that, unlike financial investment, human capital investment has decreasing returns to scale, implying an optimal amount of investment in children; unconstrained parents can always borrow enough to achieve this optimal investment, so greater financial wealth has no impact.

Moving beyond the NBL case ($\gamma = 1$ and $b = 0$), we derive the following useful comparative statics that will permeate the remainder of the analysis.

Lemma 2. *For a borrowing-constrained parent—i.e., with $(a, z, h, \varepsilon) \in int(\mathcal{B})$—consumption and investment respond to R and a as follows:*

$$\frac{\partial c_b}{\partial R} = \frac{-b x_b^{\frac{1-\alpha}{\alpha}}}{(\alpha\beta z' h^{\zeta})^{\frac{1}{\gamma}}(1 + \frac{1-\alpha}{\gamma}\frac{c_b}{x_b}) + \alpha z' h^{\zeta} x_b^{\frac{(1-\gamma)(1-\alpha)}{\gamma}}} \quad \textit{and} \quad \frac{\partial x_b}{\partial R} = -\frac{\partial c_b}{\partial R}$$

$$\frac{\partial c_b}{\partial a} = \frac{(\alpha\beta z' h^{\zeta})^{\frac{1}{\gamma}}}{(\alpha\beta z' h^{\zeta})^{\frac{1}{\gamma}}(1 + \frac{1-\alpha}{\gamma}\frac{c_b}{x_b}) + \alpha z' h^{\zeta} x_b^{\frac{(1-\gamma)(1-\alpha)}{\gamma}}} \quad \textit{and} \quad \frac{\partial x_b}{\partial a} = 1 - \frac{\partial c_b}{\partial a}.$$

Conversely, the comparative statics for an unconstrained parent are

$$\frac{\partial \log c^*}{\partial \log R} = \frac{(\beta R)^{1/\gamma}}{R + (\beta R)^{1/\gamma}}(1 - \gamma^{-1}) - \frac{z' g(x^*)/c^*}{R + (\beta R)^{1/\gamma}} \quad \textit{and} \quad \frac{\partial \log x^*}{\partial \log R} = -\frac{1}{1-\alpha}$$

$$\frac{\partial c^*}{\partial a} = \frac{R}{R + (\beta R)^{1/\gamma}} \quad \textit{and} \quad \frac{\partial x^*}{\partial a} = 0.$$

Before proceeding, we discuss the additional economic forces arising due to $\gamma \neq 1$ and $b > 0$. For unconstrained parents, higher returns R incentivize savings (substitution effect) but also increase future income from existing savings, which then incentivizes consumption (income effect). The balance of these effects is captured by the term $\frac{(\beta R)^{1/\gamma}}{R+(\beta R)^{1/\gamma}}(1 - \gamma^{-1})$, which clearly vanishes when $\gamma = 1$. In addition to this, there is still the previous log utility effect that higher R reduces the present value of child income, which reduces parental consumption through the term $-\frac{z' g(x^*)/c^*}{R+(\beta R)^{1/\gamma}}$.

For constrained parents, who have $c_b + x_b = a + b + y$, higher rates R do not move the borrowing limit, nor labor income. The only possible effect is to shift resources from consumption to investment, or vice versa (i.e., $\frac{\partial(c_b + x_b)}{\partial R} = 0$). But here with $b > 0$, c_b falls and x_b rises due to a balance-sheet effect: higher R makes borrowers' plan more expensive, which reduces expected child consumption; parents try to offset this ex-ante by consuming less and investing more. The effects of R on c_b and x_b are both of order $O(b)$ because the size of the balance-sheet effect of R is proportional to the amount of borrowing, namely b.

The critical takeaway from Lemma 2 is that $\frac{\partial}{\partial R} x^* < 0 \leq \frac{\partial}{\partial R} x_b$. As R falls, unconstrained parents invest more in their children while constrained parents invest less. A gap opens between skills of children, depending on whether their parents happened to be constrained or not. Since parents with lower income and wealth are more likely to be constrained, the skill gap in children depends on parental gaps in income and wealth. We label this *tilting*, namely the tendency for income- and wealth-rich families to disproportionately raise human capital investment in low interest rates environments.

Tilting in response to a fall in R requires borrowing constraints. In a fully unconstrained model ($b = \infty$), there are no cross-sectional differences in investment responses to interest rates:

$$\frac{\partial^2 \log x^*}{\partial a \partial R} = \frac{\partial^2 \log x^*}{\partial z \partial R} = \frac{\partial^2 \log x^*}{\partial h \partial R} = \frac{\partial^2 \log x^*}{\partial \varepsilon \partial R} = 0.$$

Low interest rates raise investment, but proportionally across the income and wealth distribution.

In the data, we will often examine the investment-to-consumption ratio (x/c), because this measure purges unmodeled effects that may apply to all types of expenditures. Indeed, a simple corollary of Lemma 2 is that x^*/c^* and x_b/c_b move in opposite directions, in response to R changes (as long as γ is not too small). Tilting manifests as a steeper slope in the association between x/c and c during low-R regimes.[4] But strictly speaking, the implications of our story depend on differential dynamics of x^* and x_b alone. To ensure that our effects are driven by x rather than c, we also explore empirically the degree of tilting in investment-to-income (x/y).

When we examine educational investment empirically, it will look like a "luxury good" in the sense that x/c is increasing in c in all surveys. Our model does not make a clear qualitative prediction on this fact. For example, Lemma 2 shows that $\frac{\partial \log(x^*/c^*)}{\partial a} < 0$ and $\frac{\partial c^*}{\partial a} > 0$, which would seem at odds with our evidence. On the other hand, one can easily show that both x^*/c^* and c^* are increasing in parental skills z and h. As long as z and h are more powerful drivers of decision-making than a, the latter effect will dominate and our model could reproduce the positive association between x/c and c. We explore this issue in Appendix B.[5]

[4] Our empirical measurement sorts households by parental consumption, which is motivated by standard permanent income theory; furthermore, it is notoriously difficult to determine whether or not a household is financially constrained.

[5] Appendix B concludes that x^*/c^* is in fact likely to be increasing in c^*. Basically, z and h modulate the relationship between x^*/c^* and c^* more strongly than a, especially if a is large; intuitively, rich parents do not base their decision-making on financial assets. The theoretical prediction that the impact of a on x^*/c^* and c^* decreases with the level of a—while the impacts of z and h do not vanish—also predicts a convex association between x^*/c^* and c^*, which we find empirically.

1.2 Implications of tilting for inequality and mobility

What are the implications of human capital tilting? We will show below that tilting implies a rise in labor income inequality, a decrease in intergenerational income mobility, and an increase in labor's share of income for relatively wealthier households (the "working rich"). All of these effects are empirically supported. We define the following proxy measures:

$$\begin{aligned}
\text{(inequality)} \quad & \mathcal{I}(R) := \mathrm{Var}[\log(y')] \\
\text{(immobility)} \quad & \mathcal{M}(R) := \frac{\mathrm{Cov}[d\log(y'), d\log(y)]}{\mathrm{Var}[d\log(y)]} \\
\text{(working rich)} \quad & \mathcal{W}(R) := \lim_{\iota\to\infty} \mathbb{E}[\frac{y'}{y'+a'} \mid y' > \iota].
\end{aligned}$$

Income variance is the simplest comprehensive dispersion/inequality measure. Looking at variance emphasizes how our model and mechanism are less about extreme income concentration and more about income inequality more broadly throughout the distribution.[6]

The measures of mobility and working rich are slightly more nuanced. The immobility measure, commonly called the "intergenerational elasticity of income" (IGE), is proxied by a regression coefficient of log child labor income on log parent labor income. Such a regression aims to capture how much a child's labor income depends on her parent's; a larger value signals higher inter-generational income persistence. The working rich measure looks at children's ratio of labor income to total income (proxied by labor income plus financial assets, since the model is static and children cannot earn capital income), among high-income children. For analytical tractability, we examine the limiting case where the child income is infinitely high.

Our proxy measures are population-level statistics that embed many types of effects. There may be differential effects for unconstrained and borrowing-constrained parents; several competing effects are simultaneously present even within groups; and the distributions of exogenous variables (a, z, h, ε) all matter. Nevertheless, in the special case of the NBL economy, we can obtain the following clean result. In this proposition, we hold fixed the set $\mathcal{B}$. Afterward, we will also analyze the effect of R on $\mathcal{B}$.

Proposition 1. *Consider the NBL economy ($b = 0$, $\gamma = 1$). Holding fixed the set $\mathcal{B}$ of borrowing-constrained parents, income inequality, intergenerational immobility, and labor's share*

[6] For example, Proposition C.1 in Appendix C shows that the Pareto tail of the income distribution is independent of R. While it is true that top percentile income shares have risen, this model is not designed to address those changes.

of income for high-income households all increase as R falls, in the following sense:

$$\frac{d}{dR}\mathcal{I}(R) < 0; \quad \frac{d}{dR}\mathcal{M}(R) < 0; \quad \textit{and} \quad \frac{d}{dR}\mathcal{W}(R) < 0.$$

Conversely, if parents' optimal investment x is held fixed (cannot change with R), then $\frac{d}{dR}\mathcal{I}(R)$ and $\frac{d}{dR}\mathcal{M}(R)$ vanish and $\frac{d}{dR}\mathcal{W}(R)$ shrinks toward zero.

These implications rely on human capital tilting. Indeed, the last line of Proposition 1 clarifies how all of our implications will either vanish or shrink without endogenous responses of x to R.

The derivation of Proposition 1 holds fixed the set $\mathcal{B}$ of borrowing-constrained parents. But the set $\mathcal{B}$ will also respond: a fall in R induces more parental consumption and investment, which leads to an equilibrium in which a greater fraction of parents are borrowing constrained. This provides an amplifying force on inequality and immobility, because borrowing-constrained parents underinvest in their children's human capital.

Proposition 2. *We have $\frac{d}{dR}\mathbb{E}\mathbf{1}_{\mathcal{B}} < 0$.*

1.3 Efficiency

We examine welfare to illustrate how human capital tilting can be inefficient. Of course, with borrowing constraints, it is easy to understand that there must be some inefficiency. However, we will go a step further and show that tilting exacerbates the existing inefficiency. We will also elaborate on the channels through a simple decomposition.

Define the utilitarian welfare measure

$$\Omega := \int_{a,z,h,\varepsilon} \Big[u(c) + \beta u(\mathbb{E}[c'])\Big] dF d\Xi. \tag{7}$$

We write Ω^{FB} to denote the "first-best" welfare that would arise in an economy without borrowing constraints (i.e., $b = \infty$), while Ω^{SB} denotes "second-best" welfare for the economy with constraints. Consider giving τ fraction of their lifetime consumption to borrowing-constrained families in the second-best world. Denote the welfare in this experiment Ω^{SB}_{τ}, which is

$$\Omega^{SB}_{\tau} := \int_{\mathcal{U}} \Big[u(c) + \beta u(\mathbb{E}[c'])\Big] dF d\Xi + \int_{\mathcal{B}} \Big[u((1+\tau)c) + \beta u((1+\tau)\mathbb{E}[c'])\Big] dF d\Xi.$$

We will use the wedge τ to define an inefficiency measure, via

$$\tau_R := \inf \{ \tau : \Omega_\tau^{SB} = \Omega^{FB} \}. \tag{8}$$

In other words, τ_R measures the percentage increase in consumption borrowing-constrained families must receive in order to ensure aggregate welfare equals first-best welfare. Higher τ_R therefore corresponds to higher inefficiency in our economy.

We are interested in how inefficiency is affected by R. Differentiating the implicit equation (8) defining τ_R, we have

$$\frac{d}{dR}\tau_R = \frac{1}{U}\Big(\frac{d}{dR}\Omega^{FB} - \frac{d}{dR}\Omega_\tau^{SB}\Big|_{\tau=\tau_R}\Big)$$

where $U := \int_{\mathcal{B}} \big[u'((1+\tau_R)c)c + \beta u'((1+\tau_R)\mathbb{E}[c'])\mathbb{E}[c']\big] dF d\Xi > 0$. So to evaluate the change in efficiency, it suffices to examine the gap $\frac{d}{dR}\Omega^{FB} - \frac{d}{dR}\Omega_\tau^{SB}\big|_{\tau=\tau_R}$. Let us further decompose this gap into a piece that holds the set of constrained parents fixed and a residual:[7]

$$\begin{aligned}\frac{d}{dR}\Omega^{FB} - \frac{d}{dR}\Omega_\tau^{SB}\Big|_{\tau=\tau_R} = \frac{d}{dR}\Omega^{FB} - \frac{d}{dR}\Omega_\tau^{SB}\Big|_{\mathcal{B}\text{ fixed},\tau=\tau_R} \\ + \frac{d}{dR}\Omega_\tau^{SB}\Big|_{\mathcal{B}\text{ fixed},\tau=\tau_R} - \frac{d}{dR}\Omega_\tau^{SB}\Big|_{\tau=\tau_R}\end{aligned}$$

Below, we will explore the first line, which holds $\mathcal{B}$ fixed and contains the key intuitions; the formal proof also considers the second line.

Consider the $b = 0$ case, in which $\frac{\partial c_b}{\partial R} = \frac{\partial x_b}{\partial R} = 0$. Consequently, the indirect utility of a constrained parent is unaffected by a change in R; only the indirect utility of unconstrained parents matters to welfare *changes*. By the envelope theorem, we have

$$\frac{d}{dR}\Omega_\tau^{SB}\Big|_{\mathcal{B}\text{ fixed},\tau=\tau_R} = \int_{\mathcal{U}} \frac{c^{-\gamma}}{R}[a+y-c-x] dF d\Xi.$$

The term $\frac{c^{-\gamma}}{R}$ is the parent's marginal utility of having an additional unit of wealth for the child, and the term $a+y-c-x$ represents the increase in child wealth from a marginal increase in R. This also helps us understand why the constrained parent utility is unaffected by R when $b = 0$: constrained parents have $a + y - c_b - c_b = -b = 0$, so there is no marginal impact of R on child wealth. Notice that $\frac{d}{dR}\Omega_\tau^{SB}\big|_{\mathcal{B}\text{ fixed}} > 0$ since unconstrained parents leave strictly positive bequests in the $b = 0$ world. Higher

[7] Term like $\frac{d}{dR}\Omega^{SB}\big|_{\mathcal{B}\text{ fixed}}$ will be computed by "differentiating under the integral sign" holding fixed the sets $\mathcal{B}$ and $\mathcal{U}$.

financial returns increase welfare by allowing savers to earn more.

Similarly, in the first-best world, we have

$$\frac{d}{dR}\Omega^{FB} = \int \frac{(c^*)^{-\gamma}}{R}[a+y-c^*-x^*]dFd\Xi.$$

Thus,

$$\frac{d}{dR}\Omega^{FB} - \frac{d}{dR}\Omega^{SB}_{\tau}\Big|_{\mathcal{B}\text{ fixed},\tau=\tau_R} = \int_{\mathcal{B}} \frac{(c^*)^{-\gamma}}{R}[a+y-c^*-x^*]dFd\Xi. \tag{9}$$

The way to read this equation is that (holding fixed $\mathcal{B}$) the increase in inefficiency consists of the welfare gain of agents in the first-best world that would be constrained in the second-best world. This is why the unconstrained policy functions (c^*, x^*) are being used in an integral over the set $\mathcal{B}$. In the $b = 0$ case, $a + y - c^* - x^* < 0$ for would-be borrowing-constrained parents, so (9) is negative. Mechanically, what is happening is that the first-best world consists of some parents with negative net wealth positions, for whom an increase in R is welfare-negative. But this masks a deeper intuition we discuss after stating the formal result.

We have the following proposition, that $\frac{d}{dR}\tau_R < 0$. In addition to the derivations above, we also show in the proof that $\left(\frac{d}{dR}\Omega^{SB}_{\tau}\big|_{\mathcal{B}\text{ fixed}} - \frac{d}{dR}\Omega^{SB}_{\tau}\right)\big|_{\tau=\tau_R} \leq 0$, because the set of unconstrained agents $\mathcal{U}$ is strictly expanding in R, combined with the fact that unconstrained agents earn higher lifetime utility.

Proposition 3. *If $b = 0$, then $\frac{d}{dR}\tau_R < 0$. Consequently, a fall in R increases inefficiency.*

There are three deep reasons why welfare falls, relative to first-best, when R falls. To understand them, let us rewrite equation (9) as follows:

$$\frac{d}{dR}\Omega^{FB} - \frac{d}{dR}\Omega^{SB}_{\tau}\Big|_{\mathcal{B}\text{ fixed},\tau=\tau_R} = \int_{\mathcal{B}} \frac{(c^*)^{-\gamma}}{R}[\underbrace{a+y-c_b-x_b}_{=0\text{ on }\mathcal{B}\text{ if }b=0}+c_b-c^*+x_b-x^*]dFd\Xi. \tag{10}$$

First, a reduction in R incentivizes investment and consumption, but constrained agents cannot take advantage of this: this effect is captured by $x_b - x^* < 0$ and $c_b - c^* < 0$ in equation (10).[8] Second, there is a covariance between marginal utility and constrained parents' under-investment and under-consumption. Poorer parents consume and invest less relative to their unconstrained optimum (i.e., $c_b - c^*$ and $x_b - x^*$ are more negative for poorer parents), and they also have higher marginal utility $(c^*)^{-\gamma}$, creating a negative covariance in equation (10). Third, when R falls, the measure of constrained parents

[8]Note that a decline in R increases consumption c^* if γ is not too high; see Lemma 2.

increases, which is welfare-negative and captured by the other element of the welfare decomposition, $\left(\frac{d}{dR}\Omega_\tau^{SB}\big|_{\mathcal{B}\text{ fixed}} - \frac{d}{dR}\Omega_\tau^{SB}\right)\big|_{\tau=\tau_R} \leq 0$.

The previous analysis was in partial equilibrium, and the shock to R is held fixed in the first-best and second-best economies. We also want to do a general equilibrium version, which allows us to consider some externalities that consumption-investment decisions have, through endogenous shifts in the rate R. To do this, we introduce a bond market clearing condition, and a wedge Φ to this condition, as follows:

$$\int [a + y - c - x] dF d\Xi = \Phi. \tag{11}$$

We think of an increase in Φ, the shock we will now analyze, as a foreign savings outflow, which induces domestic agents to save through a higher R. Importantly, the change in R can vary between the first- and second-best worlds, since constrained and unconstrained parents have differential sensitivities to R. To set up the experiment, we define Φ^{FB} and Φ^{SB} to be the initial wedges in the first-best and second-best economies, which are picked to ensure that the initial interest rates are identical, i.e., $R^{FB} = R^{SB} \equiv R_0$. Fixing the initial interest rates helps with theoretical clarity. Then, by analogy to (8), define

$$\tau_\phi := \inf\left\{\tau : \Omega_\tau^{SB}(\Phi^{SB} + \phi) = \Omega^{FB}(\Phi^{FB} + \phi)\right\}, \tag{12}$$

We then obtain the following proposition.

Proposition 4. *Suppose $b = 0$ and other parameters are such that a rise in Φ increases R in the second-best world.*[9] *Then, $\frac{d}{d\phi}\tau_\phi\big|_{\phi=0} < \left(\frac{d}{dR}\tau_R\big|_{R=R_0}\right)\frac{\partial R^{FB}}{\partial \phi}\big|_{\phi=0}$. In other words, a fall in R through general equilibrium induces a greater inefficiency than a partial equilibrium fall in R.*

One key to Proposition 4 is that interest rates are more responsive to Φ in the second-best economy than the first-best. If there is an inflow of savings (reduction in Φ), this must be offset by domestic borrowing. But because some parents are borrowing constrained, R must fall a lot to induce enough dis-saving by the unconstrained parents. Borrowing constraints effectively amplify shocks that push down R in general equilibrium. Because low rates are inefficient in our economy, this amplification by borrowing constraints is what delivers Proposition 4.

These mechanics can also be understood as an externality. Suppose borrowing constraints arise from some deeper agency friction (e.g., imperfect pledgeability, asymmetry of information). In this generic interpretation, some poor parents can always engage in inefficient behaviors (e.g., not paying back debts, shirking on effort) but do not because

[9] $\gamma \leq 1$ suffices, but is not required.

of their borrowing constraint. In this way, the potential for adverse behaviors by one poor parent is an externality on a richer parent, who must shoulder a larger burden of dis-saving when foreign inflows arrive, relative to a frictionless benchmark. Of course, poor borrowing-constrained parents do not internalize the burden their agency problem places on richer parents. In this way, while borrowing-constrained parents obviously under-invest in their children, the effect of the externality is to induce *over-investment by richer parents*. This is less conventional but intuitive upon further thought: asset-rich parents with relatively untalented children should not be investing more in their kids' education; ideally, they should be investing those resources in the children of borrowing-constrained parents.

In Appendix D.2, we consider an alternative equilibrium extension, in which the price of education is endogenous. With lower R, richer parents' human capital investments raise the price of education, which crowds out investments by poorer parents. The presence of this externality provides another angle on the statement that richer parents are *over-investing* in their children's education.

1.4 Capital tax distortions

Our framework suggests a novel reason why capital and wealth taxes are inefficient. Let us introduce a proportional tax τ on financial bequests a'. Expected child consumption is now

$$\mathbb{E}[c'] = (1-\tau)R[a+y-x-c] + z'g(x).$$

Suppose the taxes collected by government are spent on some other projects, i.e., not redistributed to the economic agents. In this extension, an increase in τ works exactly as a reduction in R, so all the previous analysis goes through. Higher capital taxes can thus create greater labor income inequality, lower intergenerational mobility, a class of "working rich", and are inefficient.

Higher capital taxes reduce financial income, inducing a substitution towards human capital investment. In a representative-agent model, the effects of this tax-induced substitution would be unclear.[10] In our heterogeneous-agent model, the substitution towards human capital is unequal: rich, unconstrained parents accumulate human capital, whereas poor, constrained parents decumulate. As we explained in equation (10), con-

[10] The typical intuition is to avoid taxing accumulable factors (Judd, 1985; Chamley, 1986), which suggests taxing labor instead of capital. But in the real world, effective units of labor are produced by accumulable skills. Furthermore, human capital may create positive externalities on growth that physical capital does not.

strained parents' under-investment in human capital is one of the key factors driving inefficiency as R falls, or equivalently here as τ rises.

Of course, the tax revenues can be redistributed in a way to mitigate the tax distortions. If the problem is that too many agents are constrained and cannot invest in human capital, the tax revenues from the asset-rich can be given to the asset-poor to relieve their financial constraints. This redistribution has a strong positive effect on constrained parents' investment, but no effect on unconstrained parents (see Lemmas 1-2). The overall welfare effect of capital taxation, plus an asset-targeted redistribution, would therefore be unclear.

1.5 Skill premium

One can imagine many possible alternative stories for human capital tilting, beyond low interest rates. We view the rise of the skill premium as a primary candidate, due to its large stature in the inequality literature. Over the past 40 years, there has been a rise in the market compensation for skills, possibly caused by technological change (e.g., skill-biased technical change), increasing returns to scale associated with globalization (e.g., superstar economics), or cheaper capital equipment driving demand for complementary skilled labor. Intuitively, a growing skill premium raises the payoff to human capital investment, moreso for richer families who have the wherewithal to do so.

To assess the validity of this claim, we modify the model in the following way. Child labor income is now

$$y' = w\varepsilon'(z'h^{\zeta}x^{\alpha})^{\chi}. \tag{13}$$

Our baseline model has $w = \chi = 1$. The interpretation of w is as the market compensation for skills, whereas χ is the elasticity of income to child human capital $h' := z'h^{\zeta}x^{\alpha}$. We can think of *skill premium growth* as an increase in w or χ or both. These two parameters have slightly different effects, as also discussed in Becker et al. (2018): an increase in w stretches the distribution of income, while an increase in χ creates additional skewness. Parents know the values of w and χ.[11]

This extension can generate a human capital tilting effect, even without the presence of borrowing constraints. For that reason, this section focuses on the behavior of

[11]Intentionally, we do not add w or χ to the parent income y. If we did, an increase in w and/or χ would have an additional income effect on parents, which they would partly consume. For unconstrained parents, this income effect would cause lower responses of x/c to skill premium increases. In this sense, modeling the skill premium increase only on children gives the model the best chance to reproduce human capital tilting.

unconstrained parents.[12] Indeed, the unconstrained investment is now given by

$$x^* = \left(\frac{\alpha\chi w (h^{\zeta} z')^{\chi}}{R}\right)^{\frac{1}{1-\alpha\chi}}. \tag{14}$$

The sensitivities of x^* to w and χ are

$$\begin{aligned}
\frac{\partial \log(x^*)}{\partial \log(w)} &= \frac{1}{1-\alpha\chi} \\
\frac{\partial \log(x^*)}{\partial \chi} &= \frac{1}{1-\alpha\chi}\Big[\log(x^*) + \frac{1}{\chi} + \zeta \log(h) + \log(z')\Big].
\end{aligned}$$

The skill-price effect (w) is proportional: all unconstrained parents increase their investment by $\frac{1}{1-\alpha\chi}$ percent in response to a 1 percent skill-price increase. Higher-investing parents (i.e., those with higher z and h), who also tend to be the parents with higher consumption, thus increase their investment by more. The skill-elasticity effect (χ) is similar in that higher z and h parents increase their investment more (but this effect is even stronger in the sense that the percentage increases in x^* are rising in z and h).

Because the rise in the US skill premium coincides with the fall in US interest rates, it is difficult to distinguish our interest rate story from a skill premium story. To help address this issue, the authors are currently in the process of investigating educational investment patterns in France, a country which has experienced a *decline* in the skill premium over the last 30 years, coincident to falling interest rates.

1.6 Wealth inequality

Another prominent secular trend is the increase in financial wealth inequality during the same time period as interest rates have fallen and labor income inequality has risen. Our baseline model cannot explain these facts jointly through an exogenous decrease in R. Indeed, Proposition C.2 in Appendix C shows that financial wealth inequality is positively associated with R.[13] But what happens if wealth inequality rises for other exogenous reasons? Could rising wealth inequality cause the human capital tilting patterns (and inequality, immobility, etc.) we document?

[12] In unreported results, we have also done a similar analysis for constrained parents. Constrained parents' educational investment responses to increases in w and/or χ are ambiguous in general. But for the extension below with an additional real asset q, constrained parents will, just like unconstrained parents, decrease their investment in q when w and χ rise.

[13] This is primarily an artifact of the static model, which implicitly assumes that financial assets are "short-term assets". With long-term assets, valuations rise as R falls, which can mechanically raise financial wealth inequality.

Here, we consider directly and exogenously altering the distribution of wealth, holding all else equal.[14] For example, one could think of rents accruing to wealthier households due to their superior investment opportunities or access to monopoly profits, which have been rising over time.

Mathematically, suppose parents have initial wealth a_0, with $\bar{a}_0 := \int a_0 dF$ denoting the average wealth. We parameterize a wealth shock by ω, defining

$$a_\omega := (1+\omega)a_0 - \omega\bar{a}_0, \tag{15}$$

which keeps the shock wealth-neutral in the aggregate, for any ω.[15] We label an increase in ω, in the sense of equation (15), *wealth-inequality growth*.

Going back to Lemma 2, we have already shown how financial assets a affect investment and consumption. Recall,

$$\frac{\partial x^*}{\partial a} = 0 \quad \text{and} \quad \frac{\partial x_b}{\partial a} > 0.$$

Financial wealth inequality creates a counterfactual pattern of human capital investment, but in a nuanced way. Wealthier parents are the ones receiving additional assets in experiment (15); poorer parents lose assets. At the same time, wealthier parents are more likely to be unconstrained. Thus, wealthier parents will respond very little to their asset increase, while poorer parents will decrease investment as they lose assets.

Formally, if $\pi(a)$ denotes the probability a parent with assets a is unconstrained, which is presumably increasing as a function of a, then the effect of wealth redistribution as in (15) is

$$\begin{aligned}\frac{d}{d\omega}\Big(\int x\delta_{a_\omega}(a)dF(a,z,h)d\Xi(\varepsilon)\Big)\Big|_{\omega=0} &= (a_0-\bar{a}_0)\Big[\pi(a_0)\frac{\partial x^*}{\partial a} + (1-\pi(a_0))\frac{\partial x_b}{\partial a}\Big] \\ &= \underbrace{(a_0-\bar{a}_0)}_{\substack{\text{wealth} \\ \text{transfer}}}\underbrace{(1-\pi(a_0))}_{\substack{\text{probability} \\ \text{constrained}}}\frac{\partial x_b}{\partial a}.\end{aligned}$$

Very low-wealth parents ($a_0 \ll \bar{a}_0$) will reduce investment for two reasons: (i) they are hit with a large negative wealth transfer; and (ii) they are very likely to be constrained,

[14] Of course, it is possible that wealth inequality causes some decline in R (Auclert and Rognlie, 2020; Mian et al., 2021, 2020). If so, then the total effect of wealth inequality will include some of what we have already analyzed in our comparative statics on R.

[15] In this sense, we have a type of mean-preserving spread. This way of changing the wealth distribution allows us to proceed with the analysis while keeping fixed F (the joint distribution of *initial* wealth and ability).

in which case investment is sensitive to wealth. High-wealth parents ($a_0 \gg \bar{a}_0$) will barely increase investment, because although they receive a large wealth inflow, they are very likely unconstrained, in which case their investment is insensitive to wealth. Near-average-wealth parents ($a_0 \approx \bar{a}_0$) will also barely change investment because they receive little wealth transfer.

In summary, wealth inequality does induce human capital tilting, but it is driven primarily by decreased investment by poorer parents. Our data suggests the opposite, that human capital tilting is driven moreso by increased investment by richer parents.

2 Empirical evidence on human capital investment

We now turn to data to see whether the Engel curve of educational expenditure has steepened as interest rate fell.

Some existing research explores related patterns. Corak (2013) and Schneider et al. (2018) have shown that the gap in the amounts spent on education has widened since the 1970s between high- and low-income households. Kornrich and Furstenberg (2013) have shown that the ratio of educational spending to income has risen more for high-income households. The relative increase in monetary spending on human-capital investment by richer households is not a substitution away from time input into monetary input, possibly in response to a divergence in the market cost of time; Ramey and Ramey (2010) have shown that time spent on children has increased more for high-income parents since the mid-1990s.

2.1 Data

We use the Consumer Expenditure Survey (CEX), for years 1960-1961, 1972-1973 and 1980-2015, to estimate the elasticity of human capital investment with respect to permanent income. The CEX allows us to estimate a proxy for the consumption share of human capital investment for households of different income levels, going back 55 years. The long time series is important, as we will examine how the elasticity gradient correlates with financial expected returns, which are slow-moving.

The main expenditure variable includes tuition, fees, books, supplies and equipment for day care centers, nursery school, other schools and college, as well as rental of books and equipment and other school-related expenses. While this measure has the benefit of being straightforward and consistent across the various CEX waves, it is only a small fraction of parents' monetary investments into children's human capital, not to mention the time investment.[16]

The main consumption variable is total expenditure. Expenditure need not equal consumption, especially given the difficult in allocating durable service flows over time. We focus on expenditure mainly for consistency across years. For example, the older waves (1960-1961 and 1972-1973) do not have enough information to impute flow of vehicle consumption.

Permanent income is estimated by instrumenting for total income with consumption. Our model is a generational model that abstracts from higher-frequency income shocks, and hence calls for sorting households by their permanent income. Following the literature on permanent income proxies, we instrument for permanent income using consumption expenditure (Modigliani and Brumberg (1954); Friedman (1957); Battistin et al. (2009); Charles et al. (2009)).

Finally, to make amounts comparable across years, we deflate using the Consumer Price Index (CPI) from BLS, so that all variables are in 2015 dollars.

2.2 Education expenditure by permanent income

Our goal is to estimate the elasticity of educational expenditure to permanent income and to estimate its change over time. In Figure 1a, we plot the ratio of educational expenditure to total expenditure against log total expenditure, for quintile groups of households sorted by the log total expenditure. In this figure, total expenditure is used directly as a proxy for permanent income. Because families' human capital investment into children depends strongly on where the children are in their life cycles, we control for this by residualizing both the ratios and the log total expenditures with age fixed effects using the age of household heads.[17] The six different lines along with their markers plot the ratios for six broad groups of years, with darker colored lines denoting later cohorts.

The first noticeable feature of Figure 1a is that education-to-consumption ratios are increasing in permanent income for all years, i.e., human capital investment is a luxury. The level of educational expenditure share may seem low, ranging from roughly 0.5% to 3.5%. This is for two reasons: (i) we only capture a small fraction of the total monetary

[16]Conceptually, many types of unmeasured monetary investments can affect and augment children's human capital: they can include expenditures on formative experiences, cognitive and emotional growth, and social networks, nutrition, health and physical fitness, to name a few.

[17]We first calculate education-to-total expenditure ratios for each household (excluding households that report zero total expenditure), then residualize both that ratio and log consumption expenditure by age, then sort by the residualized log consumption.

human capital investment, and (ii) the ratios are formed across all households regardless of the presence of school-age children. The cross-sectional spread, especially in recent years, is large. For example, in the latest years (2011-2015), the top consumption quintile households spent 3.5% of their expenditure on education, whereas the ratio is less than 0.5% for the bottom consumption quintile households.

Second, the slope is increasing over time. For example, the bottom quintile ratios across multiple years are mostly clustered around 0.5%, but the top quintile ratios display a large increase over time. An alternative way to visualize the change over time is to plot the same average educational expenditure share over time for each consumption quintile group—see Figure 2a. Each line with its markers now represents a consumption quintile group. The bottom three quintile groups' ratios are stagnant, with a slight hump-shaped pattern over time. By contrast, the top quintile group's education-to-consumption ratio increased monotonically over time, from a little above 1% in 1960 to almost 3.5% in 2011-2015.

Figures 1b and 2b instead plot ratios of educational expenditure to income, rather than total expenditure, on the y-axis. The households are still sorted into quintiles by total expenditures; this roughly corresponds to a permanent income regression where income is instrumented by consumption. We again find similar patterns.

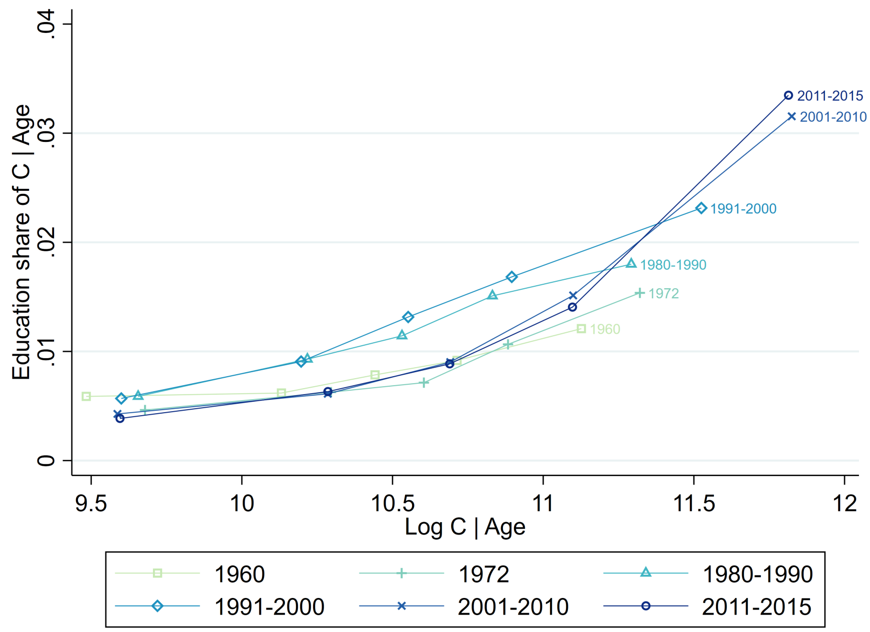
Our theory predicts human capital tilting: that the slope of human capital investment, as a function of permanent income, rises when real rates fall. In Figure 3, we therefore plot the estimated slopes and real rates together. We summarize the relationship between educational-spending share and permanent income using a semi-elasticity estimated by regressing consumption share of education on log income, instrumented using log consumption expenditure, with age and year fixed effects, for a group of years. Since the predicted relationship between the estimated slope and real rates is negative (i.e., low R leads to greater tilting), we plot $(-1) \times$ estimated slopes in Figure 3 to facilitate the comparison.

For real rates, the ideal variable to plot is an expected rate of return across all financial assets over a decade or so (or roughly the duration of returns on human capital investment), adjusted by the inflation expectation over the same horizon. Such estimates are not readily available, so we instead plot three imperfect alternatives that get different aspects of the ideal real rate variable.

The first series is the 10-year real rate, plotted in solid blue. For a risk-free financial investment, this time series is close to ideal, yet it only goes back to 1982. The second series is a 1-year real rate, plotted in solid red, which goes back further in time but is less ideal in terms of its short horizon. As expected, the real 10-year and 1-year rates exhibit similar time series movements in the years that they overlap since 1982. The third series is the earnings-to-price ratio of the US equity market, plotted in solid green. The earnings-to-price ratio is a common proxy of expected returns on the stock market and hence complements the other two series on risk-free investments.

Figure 3 presents a critical takeaway in line with our model prediction. In particular, the (negative of) estimated slopes of the human capital Engel curve tracks the real-rate proxies, decreasing first from 1960 to the 1970s, then increasing to early 1980s, and then declining in the years afterwards. The link between the human capital Engel curve and the earnings-to-price ratio of the stock market also exhibits qualitatively similar patterns. Given the limited number of directional changes in real rates over our time period Figure 3 is only a weak test, but at least the observed co-movements are consistent with our theory.

(a) Educational expenditure as a fraction of total consumption expenditure



(b) Educational expenditure as a fraction of total income

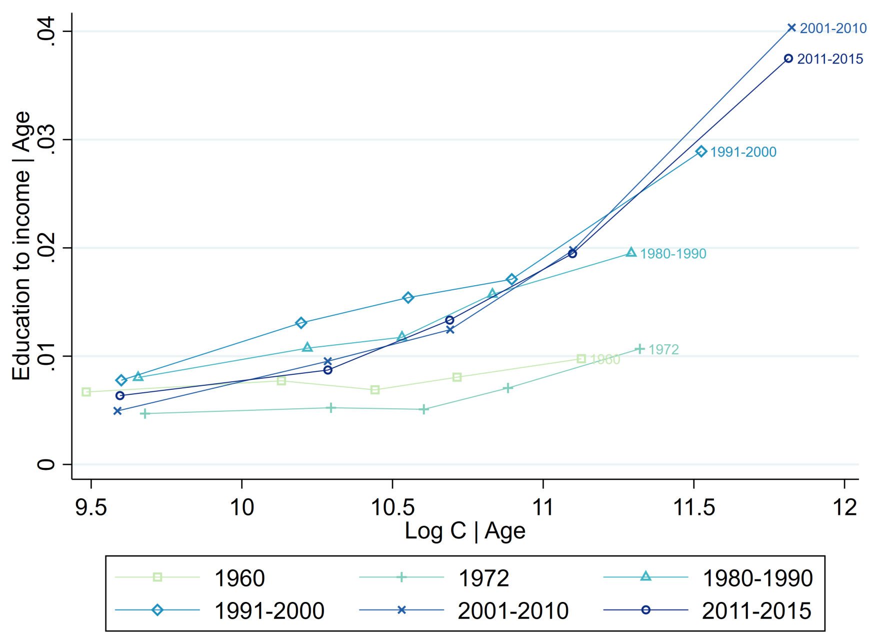
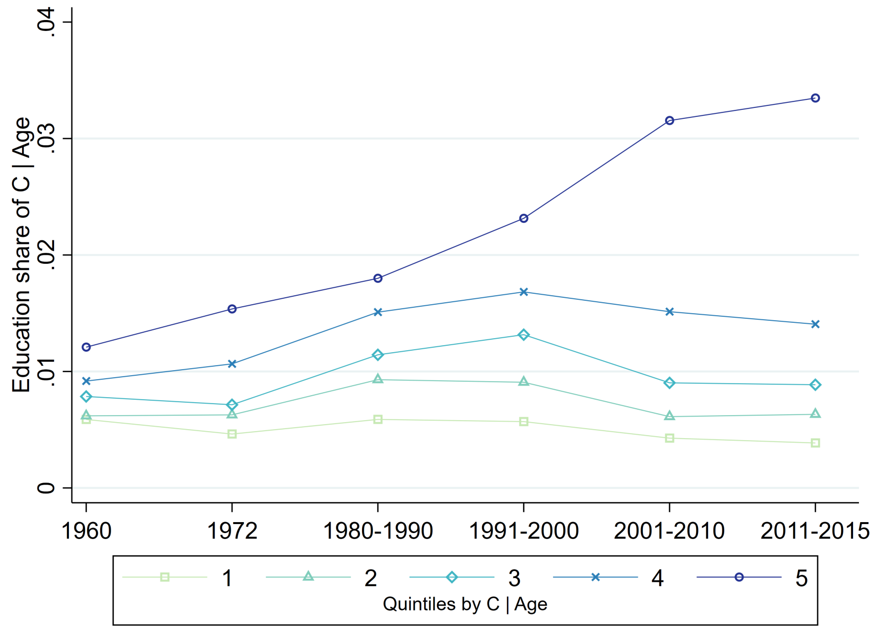


Figure 1: Education expenditure (y-axis) vs. Consumption expenditure (x-axis) for 5 consumption quintiles in the US. The top panel plots education expenditures as a fraction of total consumption expenditures. The bottom panel plots education expenditures as a fraction of income. Both the education expenditure share variables in the y-axes and the consumption variable on the x-axis are residualized with respect to age.

(a) Educational expenditure as a fraction of total consumption expenditure



(b) Educational expenditure as a fraction of total income

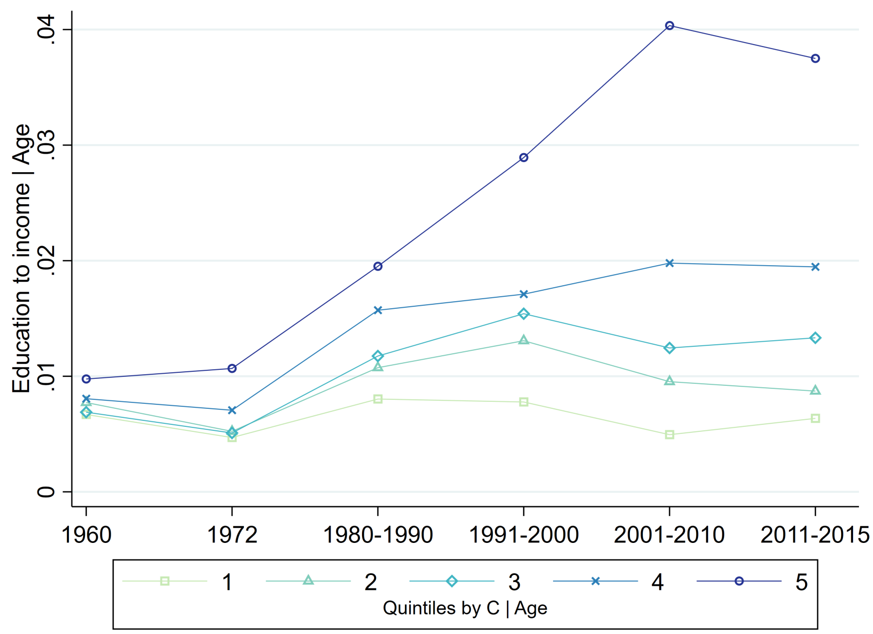


Figure 2: Education expenditure share over time. The top panel plots education expenditures as a fraction of total consumption expenditures. The bottom panel plots education expenditures as a fraction of income. Both the education expenditure share variables in the y-axes and the consumption variable used in forming the quintiles are residualized with respect to age.

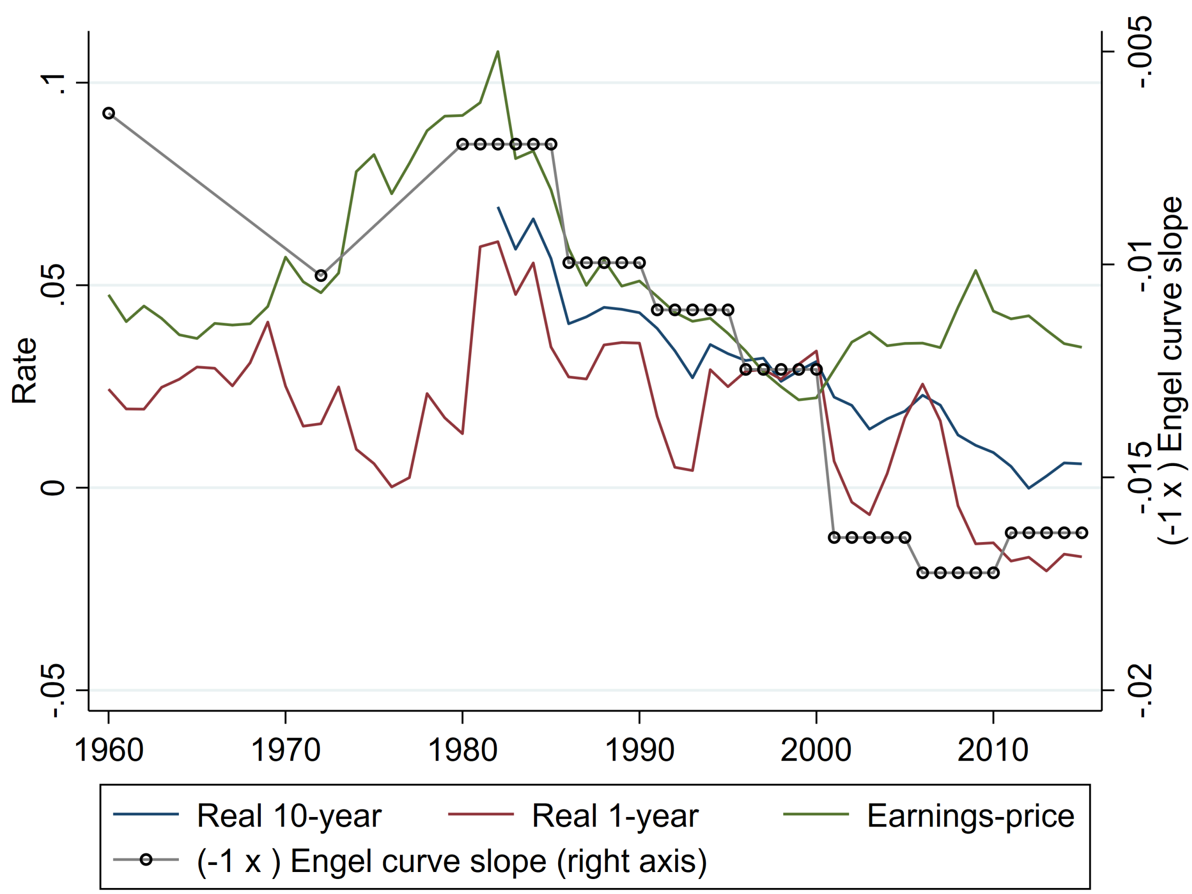


Figure 3: Education Engel slope (hollow circles, right axis) and real rates of return (solid lines, left axis). The "Engel curve slope", for a group of years, is defined as the regression coefficient from a 2-stage least squares regression of the education expenditure share on log income, with log income instrumented by log total expenditure, and including with age and year fixed effects. The figure plots the negative of this 2SLS regression coefficient. The "Real 10-year" (solid blue line) corresponds to the 10-year real Treasury yield from the Federal Reserve Bank of Cleveland, retrieved from FRED. The "Real 1-year" (solid red line) corresponds to the difference between the yield on 1-year Treasury bills, retrieved from the Macrohistory database (`https://www.macrohistory.net/database/`) constructed in Jordà et al. (2019), and the contemporaneous 1-year inflation expectation, retrieved from the Ludvigson Surveys. The "Earnings-price" (solid green line) corresponds to the earnings-to-price ratio on the S&P500 stock index, retrieved from the database (`http://www.econ.yale.edu/~shiller/data.htm`) constructed in Shiller (2000). Earnings are the sum of the last four quarters of earnings, summed over all firms in the index.

3 Quantitative magnitudes

Although our model is purposefully simple, we propose a numerical illustration to get a rough sense of effect magnitudes. In particular, we will first consider a calibration with a high interest rate that roughly matches some data from around 1980. We refer to this as the baseline calibration. Then, holding fixed the rest of the calibration, we will consider a lower interest rate and estimate the effects on human capital tilting, labor income inequality, intergenerational mobility, and the prevalence of the working rich. Conceptually, this exercise estimates how much the changes in tilting, inequality, immobility, and top labor shares can be attributed to a fall in rates between 1980 and 2010.

3.1 Calibration

Let us discuss the baseline calibration. First, parental and child labor income are given by the expressions $y = wzh\varepsilon$ and $y' = wz'h'\varepsilon'$, respectively. Here, we are allowing for an explicit wage rate w, which is chosen in order to obtain an average parental income of 40,000 per year.

The parental skill variables (z, h, ε) are drawn from a multivariate lognormal distribution, with unit means, zero correlations, and standard deviations (of the logs of these skill variables) of 0.22, 0.44, and 0.49. These standard deviations are chosen so that idiosyncratic shock ε explains half the variance of parental income, h explains 40%, z explains 10%, and so that the parental labor income Gini coefficient is 0.375 (this is approximately the value for the US in 1980).

Recall the child human capital is formed according to the production function $h' = \theta h^{\zeta} x^{\alpha}$. We calibrate $\theta = 0.07$, $\alpha = 0.35$, and $\zeta = 0.30$ in order to obtain the following in the baseline: a median child income of approximately 50,000; a top 10 percent child share of total income of around 35% (the US value in 1980, see Piketty and Saez, 2003); and a log-log IGE of approximately 0.3 (approximately the US value for cohorts born in 1955, see Davis and Mazumder, 2017). The child exogenous skill is inherited from the parents' according to $z' = z^{\psi}$, with $\psi = 0.10$ picked to reflect the relatively small estimates of genetic heritability of income-relevant talents (Bowles and Gintis, 2002). The child idiosyncratic shock ε' is drawn from the same distribution as the parent ε.

Parental financial assets a are drawn from a lognormal distribution that has a mean of 100,000, a standard deviation calibrated to match a wealth Gini of 0.5 (somewhat lower than the 1980 US wealth Gini of 0.75), and correlations of $\text{corr}[\log(a), \log(z)] = 0.1$ and $\text{corr}[\log(a), \log(h)] = 0.4$. The somewhat lower dispersion in wealth calibrated here is meant to capture the fact that assets in our model are liquid while many assets in the real world, especially for the richest families, are not. (That said, most of our results are robust to having a much wider dispersion of parental assets, with the exception that we struggle to match the 1970s slope of x/c against c in such a calibration.) The choice of correlations is so that $\log(z), \log(h)$ have the same correlations with $\log(y)$ as with $\log(a)$. For simplicity, we do not allow net borrowing, $b = 0$.

The annual discount factor is set to the relatively standard value of $\beta_{\text{annual}} = 0.95$. The baseline EIS is set to $\gamma^{-1} = 0.2$, a relatively low value on which we will perform some sensitivity analysis. The baseline annual real interest rate is set to $R_{\text{annual}} = 1.04$, in line with the "rstar" estimated by Laubach and Williams (2003); in the late 1970s and early 1980s, the long term real rate was arguably much higher, but we take a conservative approach given the difficulty in measuring inflation expectations.

Finally, in interpreting our model, we must recognize that a period our model is a "generation." Thus, we compound our discount factor and interest rate to reflect a generation of 20 years: $\beta = \beta_{\text{annual}}^{20} = 0.3585$ and $R = R_{\text{annual}}^{20} = 2.19$. A summary of our parameter choices are contained in Table 1 below.

Parameter	Description	Value
γ^{-1}	elasticity of intertemporal substitution	0.20
β_{annual}	Annual discount factor	0.95
α	HC prod. elasticity to parent investment	0.35
ψ	Exogenous skill persistence	0.10
ζ	HC prod. elasticity to parent HC	0.30
θ	HC prod. TFP	0.07
w	Wage rate	$\approx$ 40,000
b	Borrowing limit	0
R_{annual}	Annual real interest rate	1.04

Table 1: Key parameter values for the model.

3.2 Baseline results

To compute aggregate and distributional statistics, we simulate 1 million households from our model. Table 2 shows some model-implied statistics for human capital tilting, labor income inequality, intergenerational income mobility, and working-rich prevalence. Overall, the model does a reasonable job of generating relatively realistic child income distribution (panel B), levels of intergenerational mobility (panel C), and labor share of income across the distribution (panel D). Moreover, the investment-to-consumption ratio is relatively flat cross the parental income distribution (panel A), in line with the data on educational expenditure shares in 1980.

Statistic	Value ($R_{\text{annual}} = 1.04$)	Value ($R_{\text{annual}} = 1.01$)
Fraction constrained	39.2%	70.0%
A. Tilting Measures: x/c ratios at various parental labor income quantiles		
$\mathbb{E}[\frac{x}{c} \mid y = y_{0.10}^{\text{quantile}}]$	0.10	0.13
$\mathbb{E}[\frac{x}{c} \mid y = y_{0.25}^{\text{quantile}}]$	0.10	0.14
$\mathbb{E}[\frac{x}{c} \mid y = y_{0.50}^{\text{quantile}}]$	0.11	0.16
$\mathbb{E}[\frac{x}{c} \mid y = y_{0.75}^{\text{quantile}}]$	0.11	0.17
$\mathbb{E}[\frac{x}{c} \mid y = y_{0.90}^{\text{quantile}}]$	0.10	0.18
B. Inequality Measures: distribution of child labor income		
$(y')_{0.10}^{\text{quantile}}$	26,810	28,570
$(y')_{0.25}^{\text{quantile}}$	38,839	43,104
$(y')_{0.50}^{\text{quantile}}$	57,601	66,856
$(y')_{0.75}^{\text{quantile}}$	84,587	101,761
$(y')_{0.90}^{\text{quantile}}$	118,542	146,413
$\log(\frac{\text{Var}[\log y']}{\text{Var}[\log y]})$	-0.34	-0.16
Top 10pct share: $\frac{\mathbb{E}[y' \mathbf{1}\{y' \geq (y')_{0.90}^{\text{quantile}}\}]}{\mathbb{E}[y']}$	32.2%	35.5%
C. Mobility Measures: relative and absolute intergenerational dependence		
log-log IGE	0.324	0.401
$\mathbb{E}[\log(y') \mid y = y_{0.75}^{\text{quantile}}] - \mathbb{E}[\log(y') \mid y = y_{0.25}^{\text{quantile}}]$	0.33	0.39
$\mathbb{E}[\log(y') \mid y = y_{0.90}^{\text{quantile}}] - \mathbb{E}[\log(y') \mid y = y_{0.10}^{\text{quantile}}]$	0.58	0.73
D. Working-Rich Measures: labor share at various labor income quantiles		
$\mathbb{E}[\frac{y'}{y' + R_{\text{annual}} a'} \mid y' = (y')_{0.10}^{\text{quantile}}]$	0.979	0.998
$\mathbb{E}[\frac{y'}{y' + R_{\text{annual}} a'} \mid y' = (y')_{0.25}^{\text{quantile}}]$	0.925	0.990
$\mathbb{E}[\frac{y'}{y' + R_{\text{annual}} a'} \mid y' = (y')_{0.50}^{\text{quantile}}]$	0.789	0.957
$\mathbb{E}[\frac{y'}{y' + R_{\text{annual}} a'} \mid y' = (y')_{0.75}^{\text{quantile}}]$	0.638	0.868
$\mathbb{E}[\frac{y'}{y' + R_{\text{annual}} a'} \mid y' = (y')_{0.90}^{\text{quantile}}]$	0.499	0.748

Table 2: Key statistics for the baseline model calibration with $R_{\text{annual}} = 1.04$ and the counterfactual experiment with $R_{\text{annual}} = 1.01$. Other parameters are in Table 1.

Now, we do an experiment where we reduce the interest rate from $R_{\text{annual}} = 1.04$ to

$R_{\text{annual}} = 1.01$. The results are in the final column of Table 2. With the lower real interest rate, there is a significant amount of human capital tilting: investment-to-consumption x/c increases from 0.13 at the 10th percentile of parent labor income to 0.18 at the 90th percentile (panel A). This tilting drives significant inequality in terms of income distribution variance, although only a moderate change in the top 10th percentile of labor income (panel B). (The latter is somewhat to be expected, given the result of Proposition C.1 in the appendix, which shows that the Pareto tail of income in our model is unaffected by R.) All measures of intergenerational persistence are significantly higher (panel C). And finally, the labor share increases across the distribution, particularly at higher incomes, capturing the working rich phenomenon (panel D).

Relative to the data, our counterfactual exercise can account for a large degree of changes in inequality and mobility. For inequality, the cross-sectional variance in log labor incomes rises by 18% due to our mechanism: if we compare the economy with low versus high rates, the log change in child labor income variance is $\log[\mathcal{I}(1.01)] - \log[\mathcal{I}(1.04)] = 0.18$. For mobility, the log-log inter-generational elasticity (IGE) for labor incomes rises by 7.7%: if we compare the low- and high-rate economies, the change in child-on-parent log income regression coefficients is $\log[\mathcal{M}(1.01)] - \log[\mathcal{M}(1.04)] = 0.077$.

Let us mention two obvious limitations of the current model, which can be seen in the quantitative results above. First, Table 2 makes it seem like low interest rates cause the labor share to increase in aggregate, in contrast to the time series data in the US. However, this effect is due to the fact that we are essentially holding aggregate physical capital fixed. In reality, as interest rates have fallen, there has been a massive inflow of foreign capital into the US (e.g., "global savings glut"). Appendix D.1 shows that a fall in R, in an environment with a neoclassical production function featuring gross substitutability between labor and capital, necessarily generates a fall in the aggregate labor share, driven by capital deepening.

Second, Table 2 makes it seem like low interest rates cause labor incomes to rise across the board, in contrast to evidence in the US that the median wage has fallen between 1980 and 2010. This is an artifact of our assumption that the price of human capital investments is constant. In reality, the price of college and other educational goods have risen dramatically in this time period. Appendix D.2 endogenizes the supply of human capital investment goods, so that the increase in investment demand due to a fall in R generate a decrease in investments for poor households.

In summary, one should not take literally the result that child labor incomes and labor share rise across the board. Simple extensions of the baseline model can produce stagnancy or even declines in both average child labor incomes and the aggregate labor share. Thus, the results of Table 2 speak more to changes in cross-sectional dispersions and intergenerational persistences. Also, due to the caveats just mentioned, it is more difficult to get a trustworthy quantitative assessment on how our model performs on the rise of working rich.

3.3 The role of the EIS

One interesting aspect of our results is that they hold despite a relatively low value for the EIS ($\gamma^{-1} = 0.20$). A natural conjecture is that changes in R would produce more dramatic counterfactuals under a higher EIS, but this turns out to be false. Indeed, recall the discussion surrounding equation (1), where we argued that human capital investment is largely a portfolio choice decision: parents compare the marginal rate of return on investing in their child's human capital with the rate of return they get on financial investments. By contrast, the EIS governs intertemporal decisions, i.e., the tradeoff between consumption and all types of investment (including both financial and human). In the model, we rerun our counterfactual experiment comparing a low-R to a high-R economy, at different values of the EIS γ^{-1}. The results are in Table 3, which show that higher EIS actually reduces the effect of low R on tilting, inequality, and mobility. The reason a high EIS increases the sensitivity of the working rich to interest rates not because of the human capital investment margin but rather the financial savings margin: low R causes significantly more consumption and less bequests with high EIS, which means that child income is moreso comprised of labor income than capital income.

Statistic \ EIS	Change from $R_{\text{annual}} = 1.04$ to $R_{\text{annual}} = 1.01$			
	$\frac{1}{\gamma} = 0.10$	$\frac{1}{\gamma} = \mathbf{0.20}$	$\frac{1}{\gamma} = 0.50$	$\frac{1}{\gamma} = 1.00$
A. Tilting:				
$\mathbb{E}[\frac{x}{c} \mid y = y^{\text{quantile}}_{0.90}] - \mathbb{E}[\frac{x}{c} \mid y = y^{\text{quantile}}_{0.10}]$	0.06	**0.05**	0.03	0.02
B. Inequality:				
$\log(\text{Var}[\log y'])$	0.19	**0.18**	0.15	0.12
C. Mobility:				
log-log IGE	0.083	**0.077**	0.070	0.053
D. Working-Rich:				
$\mathbb{E}[\frac{y'}{y'+R_{\text{annual}}a'} \mid y' = (y')^{\text{quantile}}_{0.90}] - \mathbb{E}[\frac{y'}{y'+R_{\text{annual}}a'}]$	0.095	**0.105**	0.141	0.207

Table 3: Comparison of counterfactual experiments under different EIS calibrations (γ^{-1}). The table shows changes in some key statistics for the model with $R_{\text{annual}} = 1.01$ relative to the model with $R_{\text{annual}} = 1.04$. Other parameters are in Table 1. The second column is our benchmark calibration.

4 Conclusion

Using a simple model of the family, we connect the long-term decline in interest rates to various labor market phenomena. Lower real interest rates cause human capital tilting: richer families invest more in their children's education, while poorer families do not. Human capital tilting is not only inefficient from a welfare perspective, it also causes higher labor income inequality, an increased prevalence of the working rich, and lower intergenerational mobility. The magnitude of these effects is quite large in our benchmark calibration.

Given the central importance of the human capital tilting mechanism, we provide some novel empirical evidence on the cross-section of educational expenditure shares in the US over the last 60 years. As predicted by our model, the rich-poor expenditure share gap increases when real interest rates fall, and increases when they rise.

Appendix:
Human Capital in a Time of Low Interest Rates

Paymon Khorrami and Jung Sakong

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A Proofs

Proof of Lemma 1. Substitute $b = 0$ and $\gamma = 1$ into formulas (2), (3), (6), and $c_b = a + b + y - x_b$. $\square$

Proof of Lemma 2. Differentiate expression (6) using the implicit function theorem, and then use $c_b = a + b + y - x_b$ to obtain the results for borrowing constrained parents. Then, differentiate expressions (2)-(3) to obtain the results for unconstrained parents. $\square$

Lemma A.1. *Let $\varphi := \mathbb{E}[\mathbf{1}_{\mathcal{B}}]$ be the fraction of borrowing constrained, $\Delta_v := \mathbb{E}[\log(v) \mid \mathcal{U}] - \mathbb{E}[\log(v) \mid \mathcal{B}]$ be the between-group average difference for the log of any variable v, and $\beta^{\mathcal{G}}_{v_1,v_2} := \frac{Cov[\log(v_1),\log(v_2)|\mathcal{G}]}{Var[\log(v_2)|\mathcal{G}]}$ be the within-group regression coefficient for the logs of any variables v_1, v_2 and group $\mathcal{G} \in \{\mathcal{B},\mathcal{U}\}$. The inequality and mobility measures obey the following identities:*

$$\begin{aligned}
\mathcal{I}(R) &= (1-\varphi)Var[\log(y') \mid \mathcal{U}] + \varphi Var[\log(y') \mid \mathcal{B}] + \varphi(1-\varphi)\Delta_{y'} \\
\mathcal{M}(R) &= \Big((1-\varphi)\frac{Var[\log(y) \mid \mathcal{U}]}{Var[\log(y)]}\Big)\beta^{\mathcal{U}}_{y',y} + \Big(\varphi\frac{Var[\log(y) \mid \mathcal{B}]}{Var[\log(y)]}\Big)\beta^{\mathcal{B}}_{y',y} \\
&\quad + \Big(\varphi(1-\varphi)\frac{(\Delta_y)^2}{Var[\log(y)]}\Big)\frac{\Delta_{y'}}{\Delta_y}.
\end{aligned}$$

Proof of Lemma A.1. First, we do a general covariance decomposition with two groups $\mathcal{G}_1$ and $\mathcal{G}_2$ and two variables X and Y. We let $\mathrm{Cov}_j[X,Y] := \mathrm{Cov}[X,Y \mid \mathcal{G}_j]$ and $\mathbb{E}_j[X] := \mathbb{E}[X \mid \mathcal{G}_j]$ denote the conditional covariance and mean for group j, and $p_j := \mathbb{E}[\mathbf{1}_{\mathcal{G}_j}]$ denote the population share (probability) of group j (note $p_1 + p_2 = 1$). Then,

$$\begin{aligned}
\mathrm{Cov}[X,Y] &= p_1\mathbb{E}_1[XY] + p_2\mathbb{E}_2[XY] - (p_1\mathbb{E}_1[X] + p_2\mathbb{E}_2[X])(p_1\mathbb{E}_1[Y] + p_2\mathbb{E}_2[Y]) \\
&= p_1\mathrm{Cov}_1[X,Y] + p_2\mathrm{Cov}_2[X,Y] + p_1\mathbb{E}_1[X]\mathbb{E}_1[Y] + p_2\mathbb{E}_2[X]\mathbb{E}_2[Y] \\
&\quad - (p_1\mathbb{E}_1[X] + p_2\mathbb{E}_2[X])(p_1\mathbb{E}_1[Y] + p_2\mathbb{E}_2[Y]) \\
&= p_1\mathrm{Cov}_1[X,Y] + p_2\mathrm{Cov}_2[X,Y] + p_1p_2(\mathbb{E}_1[X] - \mathbb{E}_2[X])(\mathbb{E}_1[Y] - \mathbb{E}_2[Y]).
\end{aligned}$$

As a special case, we have $\mathrm{Var}[X] = p_1\mathrm{Var}_1[X] + p_2\mathrm{Var}_2[X] + p_1p_2(\mathbb{E}_1[X] - \mathbb{E}_1[X])^2$. Applying this variance result with X set to $\log(y')$ and $\mathcal{G}_1 = \mathcal{U}$ and $\mathcal{G}_2 = \mathcal{B}$ delivers the decomposition for $\mathcal{I}(R)$. Then, the regression betas are obtained by doing some basic manipulations to obtain

$$\frac{\mathrm{Cov}[X,Y]}{\mathrm{Var}[X]} = \left(p_1 \frac{\mathrm{Var}_1[X]}{\mathrm{Var}[X]}\right)\beta_1 + \left(p_2 \frac{\mathrm{Var}_2[X]}{\mathrm{Var}[X]}\right)\beta_2 + \left(p_1 p_2 \frac{(\mathbb{E}_1[X]-\mathbb{E}_2[X])^2}{\mathrm{Var}[X]}\right)\frac{\mathbb{E}_1[Y]-\mathbb{E}_2[Y]}{\mathbb{E}_1[X]-\mathbb{E}_2[X]},$$

where $\beta_j := \frac{\mathrm{Cov}_j[X,Y]}{\mathrm{Var}_j[X]}$ is the within-group regression coefficient. One can verify, through straightforward calculation, that the weights in the β decomposition (i.e., the objects in the large parentheses) sum up to 1. Applying this β decomposition to $X = \log(y)$ and $Y = \log(y')$ yields the result on $\mathcal{M}(R)$. $\square$

Proof of Proposition 1. As stated in the proposition, the entire proof will hold fixed the set $\mathcal{B}$ (and consequently its complement $\mathcal{U}$). In terms of the notation of Lemma A.1, we may hold fixed φ.

First, examine the inequality measure. Note that any child's income $y' = \varepsilon' z' g(x)$ can be written in log form as

$$\log(y') = \log \varepsilon' + \psi \log z + (1-\psi)\log \bar{z} + \zeta \log h + \alpha \log x. \tag{A.1}$$

Given x is the only endogenous object, and using Lemma 1, we have

$$\begin{aligned}
\frac{d}{dR}\mathrm{Var}[\log(y') \mid \mathcal{U}] &= \alpha \frac{d}{dR}\mathrm{Var}[\log(x^*) \mid \mathcal{U}] = \alpha \frac{d}{dR}\mathrm{Var}[\log(z') + \zeta \log(h) \mid \mathcal{U}] = 0\\
\frac{d}{dR}\mathrm{Var}[\log(y') \mid \mathcal{B}] &= \alpha \frac{d}{dR}\mathrm{Var}[\log(x_b) \mid \mathcal{B}] = \alpha \frac{d}{dR}\mathrm{Var}[\log(a+y) \mid \mathcal{B}] = 0\\
\frac{d}{dR}\Delta_{y'} &= \frac{d}{dR}\Big\{\mathbb{E}[\log(x^*) \mid \mathcal{U}] - \mathbb{E}[\log(x_b) \mid \mathcal{B}]\Big\} = -\frac{1}{R}\frac{1}{1-\alpha}.
\end{aligned}$$

Thus, using the decomposition of Lemma A.1 and holding fixed $\mathcal{B}$ (hence also φ), we have

$$\frac{d}{dR}\mathcal{I}(R) = \varphi(1-\varphi)\frac{d}{dR}\Delta_{y'} = -\varphi(1-\varphi)\frac{1}{R}\frac{1}{1-\alpha} < 0.$$

It is also clear from this analysis that $\mathcal{I}(R)$ would not respond to R if parental investment x was held fixed.

For the mobility measure, we again use equation (A.1), and the similar formula for parental income $\log(y) = \log(\varepsilon) + \log(z) + \log(h)$, to compute

$$\begin{aligned}
\frac{d}{dR}\beta^{\mathcal{U}}_{y',y} &\propto \frac{d}{dR}\mathrm{Cov}[\log(y'), \log(y) \mid \mathcal{U}] = \alpha\frac{d}{dR}\mathrm{Cov}[\log(x^*), \log(y) \mid \mathcal{U}] \\
&= \frac{\alpha}{1-\alpha}\frac{d}{dR}\mathrm{Cov}[\zeta\log(h) + \log(z') - \log(R), \log(\varepsilon) + \log(z) + \log(h) \mid \mathcal{U}] \\
&= \frac{\alpha}{1-\alpha}\frac{d}{dR}\Big\{\zeta\mathrm{Var}[\log(h) \mid \mathcal{U}] + \psi\mathrm{Var}[\log(z) \mid \mathcal{U}]\Big\} = 0 \\
\frac{d}{dR}\beta^{\mathcal{B}}_{y',y} &\propto \frac{d}{dR}\mathrm{Cov}[\log(y'), \log(y) \mid \mathcal{B}] = \alpha\frac{d}{dR}\mathrm{Cov}[\log(x_b), \log(y) \mid \mathcal{B}] \\
&= \alpha\frac{d}{dR}\mathrm{Cov}[\log(a+y), \log(y) \mid \mathcal{B}] = 0.
\end{aligned}$$

Thus, using the decomposition of Lemma A.1 and holding fixed $\mathcal{B}$ (hence also φ), we obtain

$$\frac{d}{dR}\mathcal{M}(R) = \varphi(1-\varphi)\frac{\Delta_y}{\mathrm{Var}[\log(y)]}\frac{d}{dR}\Delta_{y'} = \frac{\Delta_y}{\mathrm{Var}[\log(y)]}\frac{d}{dR}\mathcal{I}(R) < 0.$$

Again, it is clear that $\mathcal{M}(R)$ would not respond to R if parental investment x was held fixed.

Finally, for the working rich measure, the key is that as child income increases without bound, the probability that his/her parent is unconstrained rises to 1. Thus, compute for an unconstrained parent

$$\text{on } \mathcal{U}: \frac{y'}{y'+a'} = \frac{\varepsilon' z' g(x^*)}{\varepsilon' z' g(x^*) + R[a + y - c^* - x^*]} = \frac{\varepsilon'\frac{R}{\alpha}x^*}{\varepsilon'\frac{R}{\alpha}x^* + R[a+y-c^*-x^*]}$$

Then,

$$\begin{aligned}
\text{on } \mathcal{U}: \frac{\partial}{\partial R}\frac{y'}{y'+a'} &= \frac{\partial}{\partial R}\frac{\frac{\varepsilon'}{\alpha}x^*}{\frac{\varepsilon'}{\alpha}x^* + a + y - c^* - x^*} \\
&\propto [\frac{\varepsilon'}{\alpha}x^* + a + y - c^* - x^*]\frac{\varepsilon'}{\alpha}\frac{\partial}{\partial R}x^* - \frac{\varepsilon'}{\alpha}x^*[\frac{\varepsilon'}{\alpha}\frac{\partial}{\partial R}x^* - \frac{\partial}{\partial R}c^* - \frac{\partial}{\partial R}x^*] \\
&= [a + y - c^*]\frac{\varepsilon'}{\alpha}\frac{\partial}{\partial R}x^* + \frac{\varepsilon'}{\alpha}x^*\frac{\partial}{\partial R}c^* < 0.
\end{aligned}$$

The sign at the end is obtained using the fact that $a + y - c^* - x^* > 0$ for unconstrained parents, along with the fact that $\frac{\partial}{\partial R}x^* < 0$ and $\frac{\partial}{\partial R}c^* < 0$ from Lemma 1. This inequality holds for any a, y, ε' such that the parent is unconstrained. Consequently,

$$\frac{d}{dR}\mathcal{W}(R) = \frac{d}{dR}\lim_{\iota\to\infty}\mathbb{E}[\frac{y'}{y'+a'} \mid y' > \iota] = \frac{d}{dR}\lim_{\iota\to\infty}\mathbb{E}[\frac{y'}{y'+a'} \mid \mathcal{U}, y' > \iota] < 0.$$

Unlike the previous two measures, if x is held fixed, the working rich measure will still vary inversely to R, due to the effect of R on c^*. □

Proof of Proposition 2. Compute the following, noting that the distributions of (a, z, h, ε) are unaffected by R:

$$\begin{aligned}\frac{d}{dR}\mathbb{E}[\mathbf{1}_\mathcal{B}] &= \mathbb{E}[\frac{d}{dR}\mathbf{1}_\mathcal{B}] \\ &= \mathbb{E}\Big[\Big(\frac{\partial}{\partial(c^*+x^*)}\mathbf{1}_\mathcal{B}\Big)\frac{\partial(c^*+x^*)}{\partial R}\Big] = \frac{\partial(c^*+x^*)}{\partial R}\Big|_{\partial\mathcal{B}}\end{aligned}$$

using the fact that the "derivative" of the indicator function $\mathbf{1}_\mathcal{B} = \mathbf{1}_{\{c^*+x^*\geq a+b+y\}}$ is the Dirac delta function evaluated at the boundary of the constraint set, i.e., $\partial\mathcal{B} = \{c^*+x^* = a+b+y\}$. Next, use Lemma 2, as well as equation (3), to compute

$$\frac{\partial(c^*+x^*)}{\partial\log R} = \frac{(\beta R)^{1/\gamma}c^*}{R+(\beta R)^{1/\gamma}}(1-\gamma^{-1}) - \frac{z'g(x^*)}{R+(\beta R)^{1/\gamma}} - \frac{x^*}{1-\alpha}.$$

This expression is negative for sure if $\gamma \leq 1$. On the other hand, for the case $\gamma > 1$, we can replace c^* from equation (3) to obtain

$$\frac{\partial(c^*+x^*)}{\partial\log R} = \frac{R[a+y-x^*-c^*]+z'g(x^*)}{R+(\beta R)^{1/\gamma}}(1-\gamma^{-1}) - \frac{z'g(x^*)}{R+(\beta R)^{1/\gamma}} - \frac{x^*}{1-\alpha}.$$

Evaluating the latter expression at the boundary $\partial\mathcal{B}$ means replacing $a+y-x^*-c^* = -b$, so that

$$\begin{aligned}\frac{\partial(c^*+x^*)}{\partial\log R}\Big|_\mathcal{B} &= \frac{-Rb+z'g(x^*)}{R+(\beta R)^{1/\gamma}}(1-\gamma^{-1}) - \frac{z'g(x^*)}{R+(\beta R)^{1/\gamma}} - \frac{x^*}{1-\alpha} \\ &= \frac{-Rb}{R+(\beta R)^{1/\gamma}}(1-\gamma^{-1}) - \gamma^{-1}\frac{z'g(x^*)}{R+(\beta R)^{1/\gamma}} - \frac{x^*}{1-\alpha},\end{aligned}$$

which is negative for any $\gamma > 1$ and $b \geq 0$.

Thus, we can guarantee that for any γ, we have $\frac{\partial(c^*+x^*)}{\partial\log R}\Big|_{\partial\mathcal{B}} < 0$. This shows that $\frac{d}{dR}\mathbb{E}[\mathbf{1}_\mathcal{B}] < 0$. □

PROOF OF PROPOSITION 3. We have already shown that $\frac{d}{dR}\Omega^{FB} - \frac{d}{dR}\Omega_\tau^{SB}|_{\mathcal{B}\text{ fixed},\tau=\tau_R} < 0$ in the text. Thus, to complete the proof, we need only show that

$$\Delta_R^{\mathcal{B}} := \Big(\frac{d}{dR}\Omega_\tau^{SB}|_{\mathcal{B}\text{ fixed}} - \frac{d}{dR}\Omega_\tau^{SB}\Big)\big|_{\tau=\tau_R} \leq 0.$$

Let v_b and v^* denote the indirect utility for a constrained and unconstrained parent, respectively; these will be functions of (a, z, h, ε) as well. Note that, for any fixed (a, z, h, ε), it is clear that the unconstrained indirect utility is higher: $v^* \geq v_b$. Let $\mathcal{B}(R)$ and $\mathcal{U}(R) := \mathcal{B}(R)^c$ be the constrained and unconstrained parent sets, for a given R. Note that, for $R' > R$, we have $\mathcal{B}(R') \subset \mathcal{B}(R)$—this can be deduced from the proof of Proposition 2, which actually proves this stronger statement. This means that the sets $\mathcal{B}(R)\backslash\mathcal{B}(R')$ and $\mathcal{U}(R')\backslash\mathcal{U}(R)$ are non-empty, and in fact they are equal (i.e., the agents constrained at the lower but not higher interest rate are the same ones that are unconstrained at the higher but not lower interest rate). Using the generalized Leibniz rule, followed by the observations above, we then have

$$\begin{aligned}
\frac{d}{dR}\Omega_\tau^{SB} - \frac{d}{dR}\Omega_\tau^{SB}|_{\mathcal{B}\text{ fixed}} &= \lim_{\Delta R\searrow 0}\frac{1}{\Delta R}\Big\{\int_{\mathcal{U}(R+\Delta R)} v^* dFd\Xi + \int_{\mathcal{B}(R+\Delta R)} v_b dFd\Xi \\
&\qquad - \int_{\mathcal{U}(R)} v^* dFd\Xi - \int_{\mathcal{B}(R)} v_b dFd\Xi\Big\} \\
&= \lim_{\Delta R\searrow 0}\frac{1}{\Delta R}\Big\{\int_{\mathcal{U}(R+\Delta R)\backslash\mathcal{U}(R)} v^* dFd\Xi - \int_{\mathcal{B}(R)\backslash\mathcal{B}(R+\Delta R)} v_b dFd\Xi\Big\} \\
&= \lim_{\Delta R\searrow 0}\frac{1}{\Delta R}\int_{\mathcal{U}(R+\Delta R)\backslash\mathcal{U}(R)} (v^* - v_b) dFd\Xi \geq 0.
\end{aligned}$$

Thus, we have $\Delta_R^{\mathcal{B}} \leq 0$. □

PROOF OF PROPOSITION 4. Differentiate equation (11) to obtain

$$\frac{\partial R^{FB}}{\partial \phi} = \frac{-1}{\int [\frac{\partial c^*}{\partial R} + \frac{\partial x^*}{\partial R}] dFd\Xi}.$$

Doing the same for the second-best economy, and using the same analysis as in Proposition 3 to derive the effect of changing the set $\mathcal{B}$, we obtain

$$\begin{aligned}\frac{\partial R^{SB}}{\partial \phi} &= \frac{-1}{\int_{\mathcal{U}}[\frac{\partial c^*}{\partial R}+\frac{\partial x^*}{\partial R}]dFd\Xi + \int_{\mathcal{B}}[\frac{\partial c_b}{\partial R}+\frac{\partial x_b}{\partial R}]dFd\Xi + \lim_{\Delta R\downarrow 0}\frac{1}{\Delta R}\int_{\mathcal{U}(R+\Delta R)\setminus\mathcal{U}(R)}[c_b+x_b-c^*-x^*]dFd\Xi} \\ &= \frac{-1}{\int_{\mathcal{U}}[\frac{\partial c^*}{\partial R}+\frac{\partial x^*}{\partial R}]dFd\Xi + \lim_{\Delta R\downarrow 0}\frac{1}{\Delta R}\int_{\mathcal{U}(R+\Delta R)\setminus\mathcal{U}(R)}[c_b+x_b-c^*-x^*]dFd\Xi}.\end{aligned}$$

Note that all of these derivatives (in both the first-best and second-best expressions) are evaluated at $R = R_0$, since the two economies are assumed to begin with the same interest rate. Thus, we may take the reciprocal of these expressions and then subtract them to obtain

$$\begin{aligned}(\frac{\partial R^{SB}}{\partial \phi})^{-1} - (\frac{\partial R^{FB}}{\partial \phi})^{-1} &= \int[\frac{\partial c^*}{\partial R}+\frac{\partial x^*}{\partial R}]dFd\Xi - \int_{\mathcal{U}}[\frac{\partial c^*}{\partial R}+\frac{\partial x^*}{\partial R}]dFd\Xi \\ &\quad - \lim_{\Delta R\downarrow 0}\frac{1}{\Delta R}\int_{\mathcal{U}(R+\Delta R)\setminus\mathcal{U}(R)}[c_b+x_b-c^*-x^*]dFd\Xi \\ &= \int_{\mathcal{B}}[\frac{\partial c^*}{\partial R}+\frac{\partial x^*}{\partial R}]dFd\Xi - \lim_{\Delta R\downarrow 0}\frac{1}{\Delta R}\int_{\mathcal{U}(R+\Delta R)\setminus\mathcal{U}(R)}[c_b+x_b-c^*-x^*]dFd\Xi \\ &= \int_{\mathcal{B}}[\frac{\partial c^*}{\partial R}+\frac{\partial x^*}{\partial R}]dFd\Xi - (c_b+x_b-c^*-x^*)\big|_{\partial\mathcal{B}} \\ &= \int_{\mathcal{B}}[\frac{\partial c^*}{\partial R}+\frac{\partial x^*}{\partial R}]dFd\Xi\end{aligned}$$

Note that the term evaluated at the boundary $\partial\mathcal{B}$ equals zero because the policy functions are necessarily continuous across $\mathcal{B}$. Now, the analysis in the proof of Proposition 2 shows that $\frac{\partial(c^*+x^*)}{\partial R} < 0$ for any agent with net borrowing, which is every parent in $\mathcal{B}$. Hence,

$$(\frac{\partial R^{SB}}{\partial \phi})^{-1} < (\frac{\partial R^{FB}}{\partial \phi})^{-1},$$

which given the assumption stated in the proposition that $\frac{\partial R^{SB}}{\partial \phi} > 0$ implies

$$\frac{\partial R^{SB}}{\partial \phi} > \frac{\partial R^{FB}}{\partial \phi} > 0.$$

Next, using the implicit function theorem, we have

$$\frac{d}{d\phi}d\tau_\phi\Big|_{\phi=0} = \frac{1}{U}\Big(\frac{d}{d\phi}\Omega^{FB}(\Phi^{FB}+\phi) - \frac{d}{d\phi}\Omega^{SB}_{\tau_0}(\Phi^{SB}+\phi)\Big)\Big|_{\phi=0}$$

where $U := \int_{\mathcal{B}}\left[u'((1+\tau_0)c)c + \beta u'((1+\tau_0)\mathbb{E}[c'])\mathbb{E}[c']\right]dFd\Xi > 0$. Thus, the key term to evaluate is the welfare gap:

$$\begin{aligned}
&\Big(\frac{d}{d\phi}\Omega^{FB}(\Phi^{FB}+\phi)-\frac{d}{d\phi}\Omega^{SB}_{\tau_0}(\Phi^{SB}+\phi)\Big)\Big|_{\phi=0}\\
&=\Big[\frac{\partial\Omega^{FB}}{\partial R}\frac{\partial R^{FB}}{\partial\phi}-\frac{\partial\Omega^{SB}_{\tau_0}}{\partial R}\frac{\partial R^{SB}}{\partial\phi}\Big]\Big|_{R^{FB}=R^{SB}=R_0}\\
&=\Big[\Big(\frac{\partial\Omega^{FB}}{\partial R}-\frac{\partial\Omega^{SB}_{\tau_0}}{\partial R}\Big)\frac{\partial R^{FB}}{\partial\phi}-\frac{\partial\Omega^{SB}_{\tau_0}}{\partial R}\Big(\frac{\partial R^{SB}}{\partial\phi}-\frac{\partial R^{FB}}{\partial\phi}\Big)\Big]\Big|_{R^{FB}=R^{SB}=R_0}
\end{aligned}$$

The result of Proposition 3 was that $\frac{\partial\Omega^{FB}}{\partial R}-\frac{\partial\Omega^{SB}_{\tau_0}}{\partial R}<0$. In addition, the derivations leading up to equation (9) show that $\frac{\partial\Omega^{SB}_{\tau_0}}{\partial R}\big|_{\mathcal{B}\text{ fixed}}>0$, while the appendix derivations above for the proof of Proposition 3 show that $\frac{\partial\Omega^{SB}_{\tau_0}}{\partial R}-\frac{\partial\Omega^{SB}_{\tau_0}}{\partial R}\big|_{\mathcal{B}\text{ fixed}}\geq 0$. Combining, we thus have $\frac{\partial\Omega^{SB}_{\tau_0}}{\partial R}>0$. Finally, from the previous analysis, we have obtained $\frac{\partial R^{SB}}{\partial\phi}>\frac{\partial R^{FB}}{\partial\phi}>0$. Putting these facts together, we obtain $\frac{d}{d\phi}d\tau_\phi\big|_{\phi=0}<\frac{d}{dR}d\tau_R\big|_{R=R_0}\frac{\partial R^{FB}}{\partial\phi}\big|_{\phi=0}$. □

B Investment-to-consumption gradient

This section is dedicated to exploring the conditions under which an unconstrained household will exhibit an x/c that is increasing in c, as in the data. Initially, one may think this is difficult to obtain, because Lemma 2 shows that $\frac{\partial\log(x/c)}{\partial a}<0$ for unconstrained parents. But if other factors besides financial wealth are the primary drivers of c, then it may be possible to obtain x/c which is increasing in c. For this section, suppose there is no borrowing constraint (i.e., $b=\infty$).

By inspection of the various optimality conditions, we see that heterogeneity in parental consumption is governed by heterogeneity in (a,z,h,ε). By differentiating expressions (2) and (3), we obtain the following set of comparative statics:

$$\begin{aligned}
\frac{\partial\log c}{\partial a}&=\frac{R}{R[a+\varepsilon zh-x]+z'g(x;h)} & \frac{\partial\log x}{\partial a}&=0\\
\frac{\partial\log c}{\partial\varepsilon}&=\frac{Rzh}{R[a+\varepsilon zh-x]+z'g(x;h)} & \frac{\partial\log x}{\partial\varepsilon}&=0\\
\frac{\partial\log c}{\partial\log z}&=\frac{R\varepsilon zh+\psi z'g(x;h)}{R[a+\varepsilon zh-x]+z'g(x;h)} & \frac{\partial\log x}{\partial\log z}&=\frac{\psi}{1-\alpha}\\
\frac{\partial\log c}{\partial\log h}&=\frac{R\varepsilon zh+\zeta z'g(x;h)}{R[a+\varepsilon zh-x]+z'g(x;h)} & \frac{\partial\log x}{\partial\log h}&=\frac{\zeta}{1-\alpha}
\end{aligned}$$

Combining these results, we obtain the following x/c partial slopes against the four variables that increase parental consumption:

$$\frac{\partial \log(x/c)}{\partial a} = -\frac{R}{R[a+\varepsilon zh - x] + z'g(x;h)}$$
$$\frac{\partial \log(x/c)}{\partial \varepsilon} = -\frac{Rzh}{R[a+\varepsilon zh - x] + z'g(x;h)}$$
$$\frac{\partial \log(x/c)}{\partial \log z} = \frac{\psi}{1-\alpha} - \frac{R\varepsilon zh + \psi z'g(x;h)}{R[a+\varepsilon zh - x] + z'g(x;h)}$$
$$\frac{\partial \log(x/c)}{\partial \log h} = \frac{\zeta}{1-\alpha} - \frac{R\varepsilon zh + \zeta z'g(x;h)}{R[a+\varepsilon zh - x] + z'g(x;h)}$$

Notice that the first and second expressions are unambiguously negative. Thus, for this model to replicate the empirical result that x/c is increasing in c, parental consumption cannot be significantly determined by wealth a and transitory luck ε.

Supposing this to be true, the third and fourth expressions above become relevant. These expressions will be positive under the following conditions. The third expression is only positive if talent heritability ψ and human investment returns-to-scale α are large. Whereas returns-to-scale α are plausibly non-trivial, the genetic heritability of income-relevant talents explains only a small fraction of the persistence of income, education, and wealth (Bowles and Gintis, 2002), which suggests a smaller value of ψ. By contrast, the fourth expression is positive if the complementarities between parent and child human capital, measured by ζ, are sufficiently large. A recent literature emphasizes the quantitative importance of these parent-child complementarities (Cunha and Heckman, 2007; Caucutt and Lochner, 2020).

Finally, one can also see that for sufficiently rich parents (i.e., $a \to \infty$), both $\frac{\partial \log(x/c)}{\partial \log z}$ and $\frac{\partial \log(x/c)}{\partial \log h}$ necessarily become positive, whereas $\frac{\partial \log(x/c)}{\partial a}$ and $\frac{\partial \log(x/c)}{\partial \varepsilon}$ necessarily become negligible. Consequently, for sufficiently rich parents, it must be the case that x/c is increasing in c.

C Other inequality measures

In our baseline model, neither the thickening tail of the income distribution, nor rising wealth inequality can be explained by low R. The next two propositions formalize this statement.

Proposition C.1. *Suppose the idiosyncratic shock distribution Ξ has a right Pareto tail with tail index $\xi > 1$. Then, the distribution of children's income y' inherits a Pareto tail with index ξ.*

Proof of Proposition C.1. Recall $y' = \varepsilon' z' g(x)$. It suffices to examine an unconstrained parent asymptotically (as parental income increases without bound), since ε' is independent of all other variables, since $z' \to \infty$ implies $z \to \infty$ so that $(a, z, h, \varepsilon) \in \mathcal{U}$ asymptotically, and since x is bounded for parents in $\mathcal{B}$. Asymptotically, we thus have $x \sim [\alpha h^{\zeta} z^{\psi} \bar{z}^{1-\psi}/R]^{\frac{1}{1-\alpha}}$. Compute (where $\sim$ denotes asymptotic equality as $y_0 \to \infty$)

$$\begin{aligned}
\mathbb{P}(y' > y_0) &= \mathbb{P}(\varepsilon' z' x^{\alpha} > y_0) \\
&\sim \mathbb{P}(\varepsilon' > [h^{\zeta} z^{\psi} \bar{z}^{1-\psi}]^{-\frac{1}{1-\alpha}} [\alpha/R]^{-\frac{\alpha}{1-\alpha}} y_0) \\
&= \int \Big[1 - \Xi\Big([\alpha^{-\alpha} h^{-\zeta} z^{-\psi} \bar{z}^{-(1-\psi)} R^{\alpha}]^{\frac{1}{1-\alpha}} y_0\Big)\Big] dF \\
&\sim \int \Big([\alpha^{-\alpha} h^{-\zeta} z^{-\psi} \bar{z}^{-(1-\psi)} R^{\alpha}]^{\frac{1}{1-\alpha}} y_0\Big)^{-\xi+1} dF \\
&= [\frac{\alpha}{R}]^{\alpha(\xi-1)/(1-\alpha)} \bar{z}^{(\xi-1)(1-\psi)/(1-\alpha)} \mathbb{E}[(h^{\zeta} z^{\psi})^{(\xi-1)/(1-\alpha)}] y_0^{-\xi+1} \\
&\sim \text{constant} \times y_0^{-\xi+1}.
\end{aligned}$$

Hence, the Pareto tail is $\lim_{y_0 \to \infty} 1 - \log \mathbb{P}(y' > y_0)/\log y_0 = \xi$. □

Proposition C.2. *Holding $\mathcal{B}$ fixed, and assuming γ is not too large, the between-group wealth-inequality measure $\mathbb{E}[a' \mid \mathcal{U}] - \mathbb{E}[a' \mid \mathcal{B}]$ is increasing in R.*

Proof of Proposition C.2. Compute

$$\begin{aligned}
&\frac{d}{dR}\Big\{\mathbb{E}[a' \mid \mathcal{U}] - \mathbb{E}[a' \mid \mathcal{B}]\Big\} \\
&\quad = \mathbb{E}[a + y - c^* - x^* \mid \mathcal{U}] - \mathbb{E}[a + y - c_b - x_b \mid \mathcal{B}] \\
&\qquad - R\mathbb{E}[\frac{\partial}{\partial R}(c^* + x^*) \mid \mathcal{U}] + R\mathbb{E}[\frac{\partial}{\partial R}(c_b + x_b) \mid \mathcal{B}] \\
&\quad = \mathbb{E}[a + y - c^* - x^* \mid \mathcal{U}] + b - R\mathbb{E}[\frac{\partial}{\partial R}(c^* + x^*) \mid \mathcal{U}] + 0 \\
&\quad > -R\mathbb{E}[\frac{\partial}{\partial R}(c^* + x^*) \mid \mathcal{U}],
\end{aligned}$$

where the final inequality uses $a + y - c^* - x^* > -b$. Now, given γ is not too large, we can guarantee $\frac{\partial}{\partial R} c^* < 0$ and $\frac{\partial}{\partial R} x^* < 0$ (see the expression in Lemma 2). In that case, we obtain the result $\frac{d}{dR}\{\mathbb{E}[a' \mid \mathcal{U}] - \mathbb{E}[a' \mid \mathcal{B}]\} > 0$. □

D Other extensions

D.1 Endogenous wages

In this section, we model a neoclassical aggregate production function in order to demonstrate that a decline in R can actually increase skill premia and nevertheless reduce the labor share—both of which have occurred in the data.

Let the constant returns to scale aggregate production function be $\varphi(L, K)$, where L is labor, measured in efficiency units of human capital, and K is physical capital. The firm pays R on its rental of physical capital and w on its labor rental. Thus, children now earn $y' = \varepsilon' w z' g(x)$ in this model. The labor market clears, in the sense that (recall ε' is independent of z', h, and ε, with $\int \varepsilon' d\Xi = 1$)

$$L = \int z' g(x) dF d\Xi.$$

The capital market is perfectly elastic, with the interest rate R is given exogenously.

In this world, the firm's problem implies

$$R = \varphi_K(L/K) \quad \text{and} \quad w = \varphi_L(L/K) \quad \text{for} \quad \varphi_K' > 0,\ \varphi_L' < 0.$$

A decline in R raises wages unambiguously.

Lemma D.1. *If the economy possesses a CRS production function $\varphi(L, K)$, then an exogenous decrease in R raises w:*

$$\frac{d \log w}{d \log R} = -\frac{\sigma_K}{\sigma_L} < 0,$$

where $\sigma_L := \frac{wL}{\varphi}$ is labor's share and $\sigma_K := 1 - \sigma_L$ is capital's share.

Proof of Lemma D.1. Differentiate the two first-order conditions $\varphi_K(L/K) := \partial_K \varphi(L, K) = R$ and $\varphi_L(L/K) := \partial_L \varphi(L, K) = w$, and use Euler's theorem for homogeneous functions to obtain the equality $\frac{d \log w}{d \log R} = -\frac{\sigma_K}{\sigma_L}$. □

Consequently, through higher skill prices, the analysis of Section 1.5 applies in addition to the direct effects of falling R. Quantitatively, since $\sigma_K \approx 0.4$ and $\sigma_L \approx 0.6$, wages rise by 2/3 as much (in percentage points) as R falls. Since parents take equilibrium R and w as given, and have rational expectations, they understand that a low-R environment is associated with a high-w environment. This would amplify our previous analysis, increasing human capital investment incentives.

The logic flows through the quantity of physical capital employed. Consider that any shock that leads to lower R also expands the capital stock, for example an inflow of resources from abroad (e.g., "global savings glut"). Any increase in K raises the marginal productivity of human capital in the firm, regardless of the degree of production complementarity/substitutability between human and physical capital.

Another topic of interest is the effect of lower R on the labor share σ_L. If wages are higher, one would expect the labor share to rise. In fact, it is ambiguous, because of the endogenous increase in K.

Lemma D.2. *Denote the elasticity of substitution by* $\sigma_{KL} := \frac{d\log(K/L)}{d\log(\partial_L f/\partial_K f)}$. *In response to a fall in* R*, the labor share decreases if and only if* $\sigma_{KL} > 1$*, and in general*

$$\frac{d\log\sigma_L}{d\log R} = -\frac{\sigma_K}{\sigma_L}(1-\sigma_{KL}).$$

PROOF OF LEMMA D.2. By evaluating the definition of σ_{KL}—using the implicit function theorem, the result of Lemma D.1, and Euler's theorem for homogeneous functions—we have

$$\sigma_{KL} = \frac{d\log(K/L)}{d\log(\partial_L\varphi/\partial_K\varphi)} = \frac{\frac{d\log(K/L)}{d\log R}}{\frac{d\log(\partial_L\varphi/\partial_K\varphi)}{d\log R}} = \sigma_L\frac{d\log(\varphi_K^{-1}(R))}{d\log R}.$$

Use this result, along with Lemma D.1 in the following

$$\frac{d\log\sigma_L}{d\log R} = \sigma_K\frac{d\log\sigma_L/\sigma_K}{d\log R} = \sigma_K\Big[-\frac{d\log R}{d\log R} + \frac{d\log w}{d\log R} + \frac{d\log\varphi_K^{-1}(R)}{d\log R}\Big]$$

to obtain the desired result. □

The condition $\sigma_{KL} > 1$ says that labor and capital are more substitutable than Cobb-Douglas, which has constant labor and capital shares.

D.2 Endogenous price of education

Previously, we have considered the price of human capital investment to be exogenously fixed at 1, as with the numeraire consumption good, implicitly assuming a perfectly elastic supply curve for human capital investment goods. Here, we suppose there is an upward-sloping supply curve $S(p)$ with $S'(p) > 0$. The market-clearing condition for investments says that the total investment equals the supply:

$$\int x dF d\Xi = S(p). \tag{D.1}$$

Of course, any shock (e.g., $R \downarrow$) that leads to an increase in human capital demand will raise equilibrium p. To study this, let us focus on the NBL economy (with $\gamma = 1$ and $b = 0$) for simplicity; in this case, only unconstrained parents respond to R.

Differentiating the market clearing condition (D.1), we obtain

$$\frac{d \log p}{d \log R} = \underbrace{\left(\frac{S(p)}{pS'(p)}\right)}_{\substack{\text{supply elasticity} \\ \text{(inverse)}}} \mathbb{E}\Big[\underbrace{\left(\frac{\partial \log x^*}{\partial \log R}\right)}_{\substack{\text{response by} \\ \mathcal{U} \text{ parents}}} \underbrace{\frac{x^*}{\mathbb{E}[x]} \mathbf{1}_{\mathcal{U}}}_{\substack{\text{relative} \\ \text{investment}}} \Big] < 0.$$

If S is a constant elasticity function with elasticity ν^{-1}, then $\frac{d \log p}{d \log R} = -\frac{\nu}{1-\alpha} \frac{\mathbb{E}[x^* \mathbf{1}_{\mathcal{U}}]}{\mathbb{E}[x^* \mathbf{1}_{\mathcal{U}}] + \mathbb{E}[x_b \mathbf{1}_{\mathcal{B}}]}$.[18]

The main additional implication is a "crowding-out" effect, whereby higher prices reduce poorer parents' investments, where previously there was no effect.

Lemma D.3. *Suppose $b = 0$ and $\gamma = 1$. With endogenous prices, a fall in the interest rate R induces less investment by borrowing-constrained parents: $\frac{dx_b}{dR} > 0$.*

Proof of Lemma D.3. The new borrowing constraint is $a + y - c_b - px_b = 0$, and the new optimality condition is

$$\beta z' g'(x_b) = p \frac{u'(c_b)}{u'(z' g(x_b))}.$$

Using $b = 0$ and $\gamma = 1$, we obtain

$$x_b = \frac{1}{p} \frac{\alpha\beta}{1 + \alpha\beta}(a + y).$$

Therefore, $\frac{\partial x_b}{\partial R} = -\frac{x_b}{p} \frac{dp}{dR} > 0$. □

Rising costs of education, which reduce investment by constrained parents, has two implications. First, because under-investment is a key determinant of inefficiency (recall formula (10)), rising educational costs further reduce efficiency under borrowing constraints. Second, lower skill attainment by the constrained suggests labor income for a substantial population could actually decrease when R falls. Thus, the endogenous rise

[18] Indeed, although optimal investment by unconstrained parents is now modified to

$$x^* = \left(\frac{\alpha z' h^{\zeta}}{pR}\right)^{\frac{1}{1-\alpha}},$$

their elasticity to R is unaffected by p.

in education costs could be an important piece of the puzzle for understanding why the median wage has not increased, despite falling R and rising skill premia.

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